THE CONSTRUCTION AND USE OF A TEST MACHINE TO MEASURE DYNAMIC LOADS ON GEAR TEETH

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by

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ABSTRACT

The action of mating gear teeth is a very complex thing. Very little is known of the forces that act on gears as they mesh. It is with this in mind that a testing machine was designed, built, and used to investigate the feasibility of obtaining gear tooth load information, by the use of strain gages mounted on the teeth of a test gear. The construction and operation of the testing maching is described, followed by the results of experiments conducted. The application of the experimental results to some design considerations in gears is then discussed. A great need exists for gear loading information, and the gear testing machine has proved to be successful in the initial phases of its operation.

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ACKNOWLEDGEMENT

The construction of the gear testing maching has been a long but rewarding task. Many thanks are due to those who have helped. Thank you especially to Professor E. K. Gatcombe for his guidance and encouragement, and to Henry Perry of the Machine Facility, USNPGS, for the long hours spent in fabricating the various components of the machine.

I. INTRODUCTION

The action of mating gear teeth is a very complex thing. Much has been theorized about what takes place when one gear drives another, and indeed, much is known of the conjugate action of the involute gear, but very little is known of the forces that act on a gear as it meshes with its mating gear.

It is with this in mind that a testing maching, figure 1, was designed, built, and used to investigate the feasibility of obtaining gear tooth load information, by the use of strain gages mounted on the teeth of a test gear.

This investigation was carried out in three general phases; construction, operation and instrumentation, and testing.

The testing machine is made up of several sub units; the test unit, the main drive system, lubrication systems, hydraulic system, the base and supporting assemblies, which include all control panels, and external instrumentation. All these sub units are described in more detail in subsequent sections.

The results of the tests conducted during the experimental phase indicated an aspect of gear loading that is not generally considered. It was apparent that the contact ratio of mating gears plays a part in how the gear teeth are loaded. This aspect is not considered in any of the widely used methods of predicting dynamic loads on gear teeth.

It is hoped that this investigation is the beginning of a continuing effort utilizing the testing facilities and methods developed for this purpose.

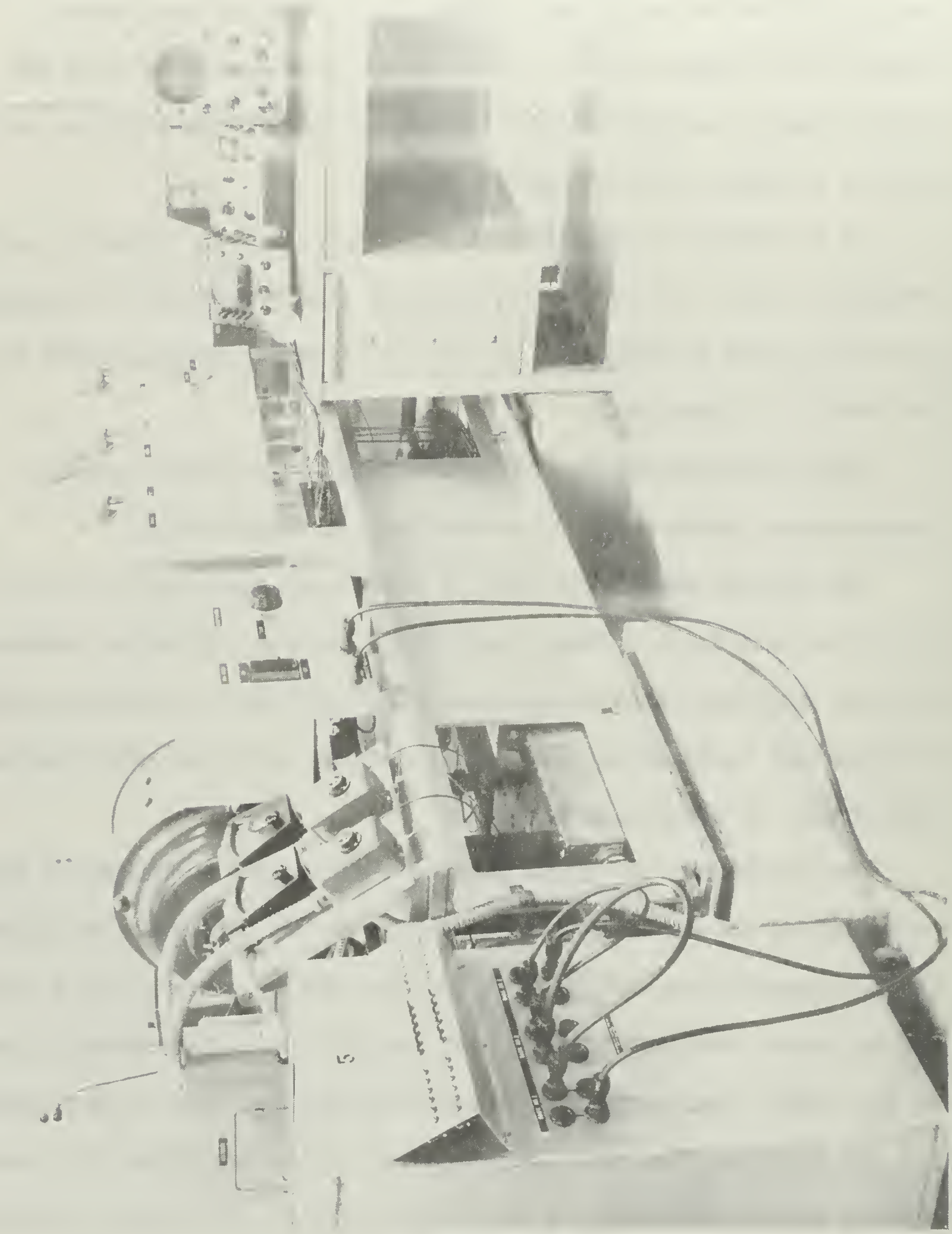


FIGURE 1. [illegible]

II. CONSTRUCTION

TEST UNIT

The test unit consists of a four square device which can be loaded hydraulically during operation, figure 2. The basic design work for this unit was done by H.J. HANSEN in 1965-66. [6]

All parts of the test unit were fabricated by the Machine Facility, United States Naval Postgraduate School, with the exception of the gears, which were obtained from commercial sources.

Minor changes were made during construction, but the basic design remains much the same as that reported by HANSEN.

The test unit is divided into two basic sections, a drive section and a test section. Two parallel shafts pass through these two sections, and each carries a gear on both ends, figure 3. The gears on the shaft ends in the test section are referred to as the test gear and the test drive gear. These two gears are straight cut spur gears. The test gear is fastened to its shaft with a steel taper pin, and the test drive gear utilizes a brass shear pin to provide overload protection.

In the drive section, the shafts carry helical spur gears, one of which is axially movable on its shaft, and shall be referred to as the drive loading gear. The other helical gear in the drive section is splined and bolted to its shaft, and cannot move axially. One helical gear is of the right hand and the other is of the left hand.

An axial displacement between these two helical gears causes a rotation of the shafts, both shafts turning in the same direction. Since the shafts are locked together at the test end by the test gears, this shaft rotation introduces a torque load into the four square system.

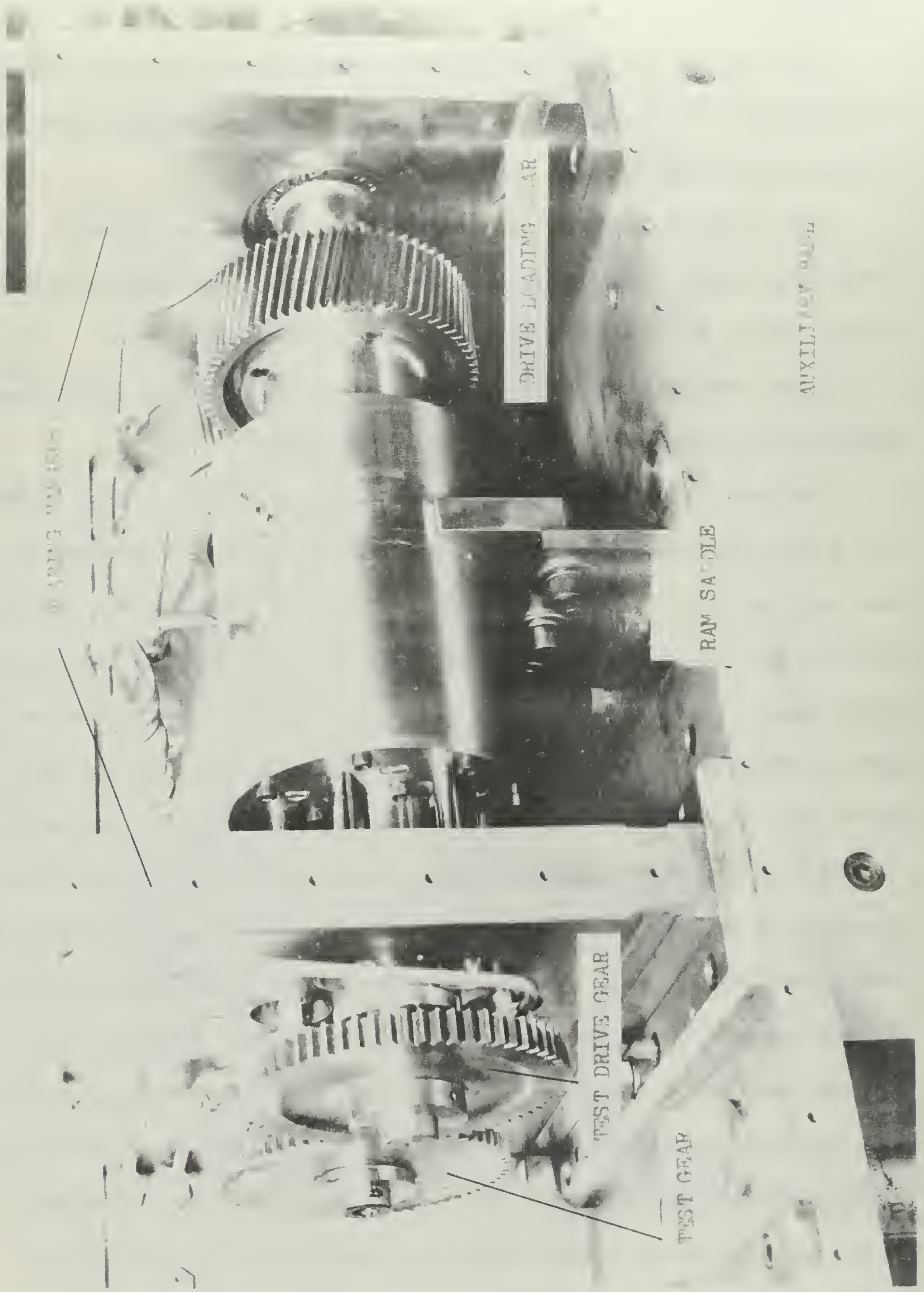


FIGURE 2. TEST UNIT

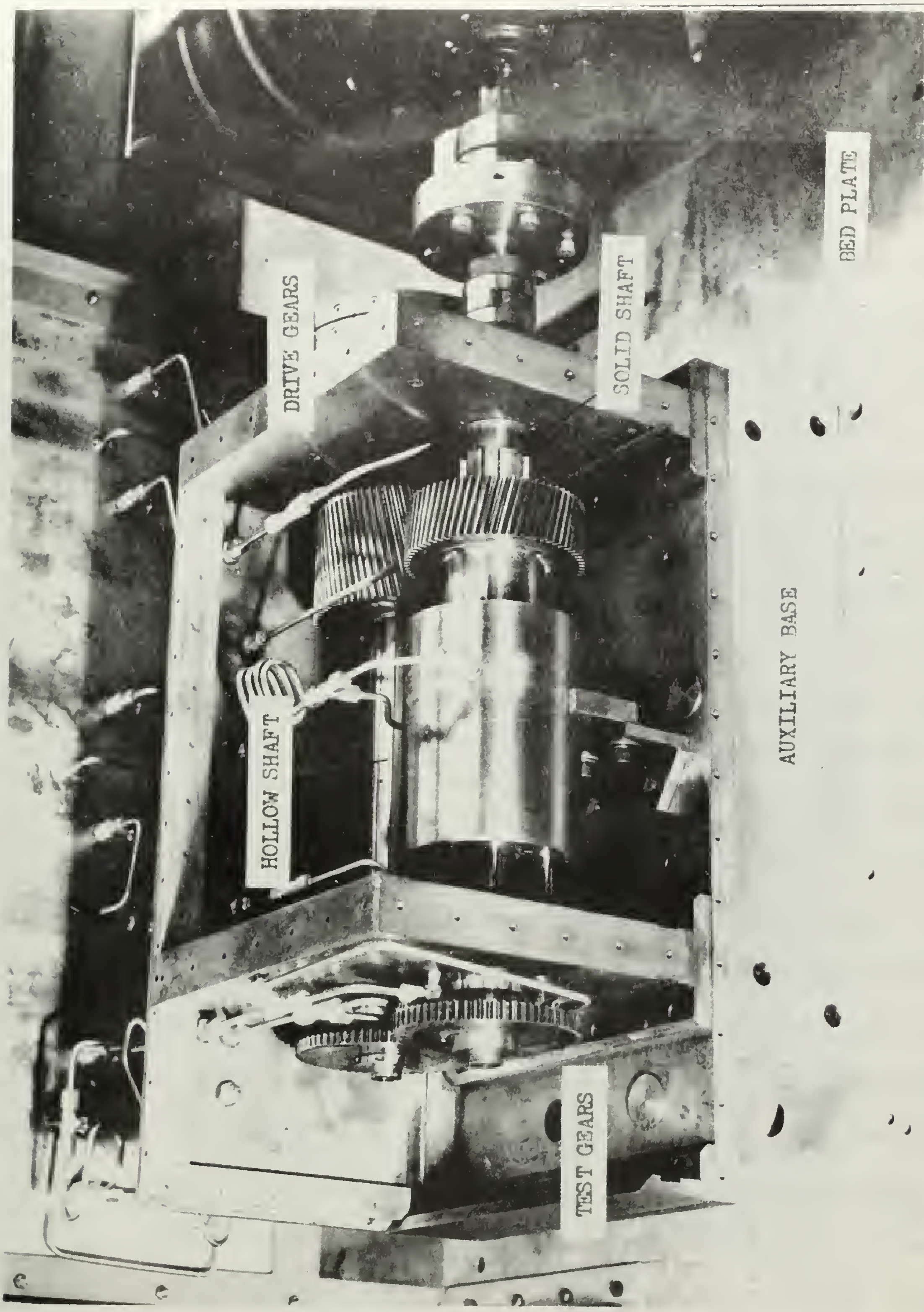


FIGURE 3. TEST UNIT SHOWING FOUR SQUARE PRINCIPLE

The axial displacement of the drive loading gear is accomplished by a hydraulic cylinder and ram assembly which can move in either direction. The ram assembly was originally supported by six piston rods at the test end bearing hanger, and the drive loading gear where it is splined to its shaft. See figure 4. In order to aid the support of the ram assembly, a brass saddle as shown in figure 4 was designed. The saddle is adjustable up and down as well as transversely. This feature is an aid to ram alignment on assembly of the test unit.

A bronze bushing was substituted for the Teflon coating inside the ram assembly. The Teflon was called for in the original design, but proved to be unsatisfactory.

Timken tapered roller bearings are used throughout the test unit. The thrust of the hydraulic loading ram is taken up by a double tapered roller bearing located inside the ram assembly.

Bearing adjusters are provided for both shafts on the drive end bearing hanger. These adjusters are in the form of a large hollow threaded device that holds the bearing cup. See figure 5. By turning the adjuster, axial movement of the bearing is effected, adjusting both bearings of one shaft simultaneously. Located on the periphery of each adjuster are a series of punch marks, and there is an index mark on the bearing hanger. Turning the adjuster through one mark corresponds to an axial movement of the adjuster of .001 inch.

The shaft carrying the test gear and the wide face helical gear is hollow, to permit electrical leads to be run to instrumentation points. This hollow shaft carries a changeable mass flywheel, shown in figure 6. This flywheel consists of a brass rim and a set of twelve segments that can be bolted to the rim to change the mass moment of

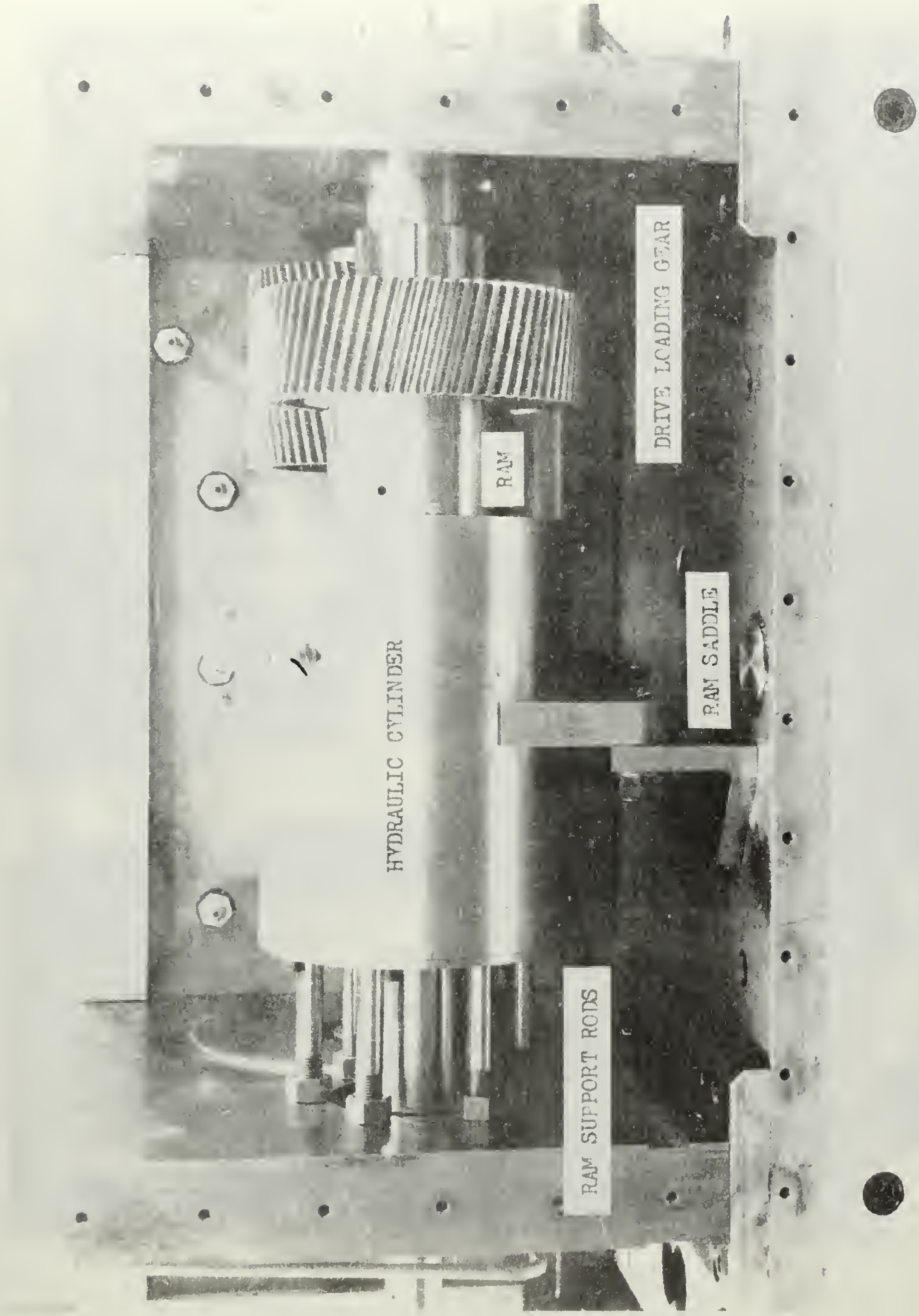


FIGURE [illegible]. LOADING RAM ASSEMBLY

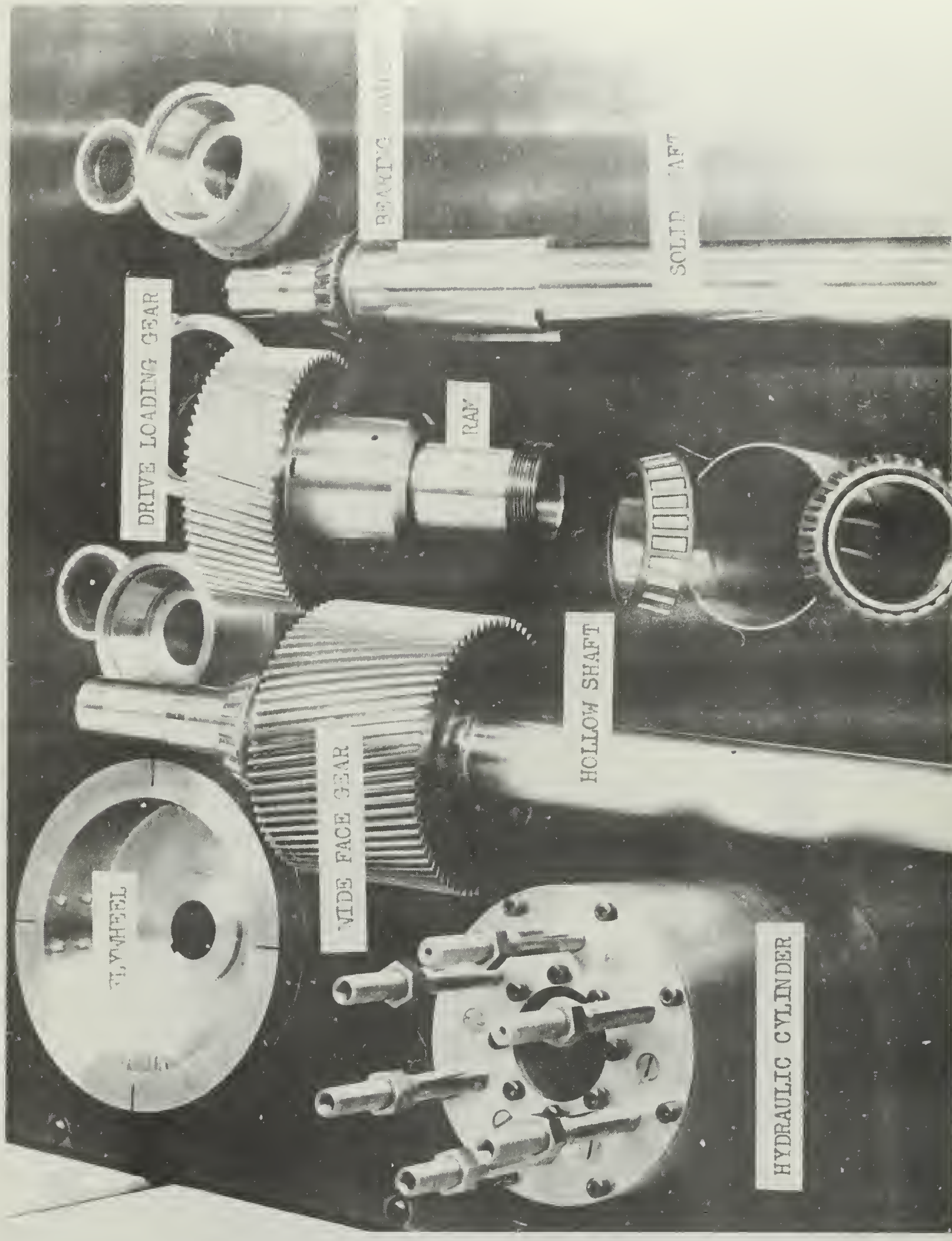


FIGURE [illegible]. TEST UNIT COMPONENTS

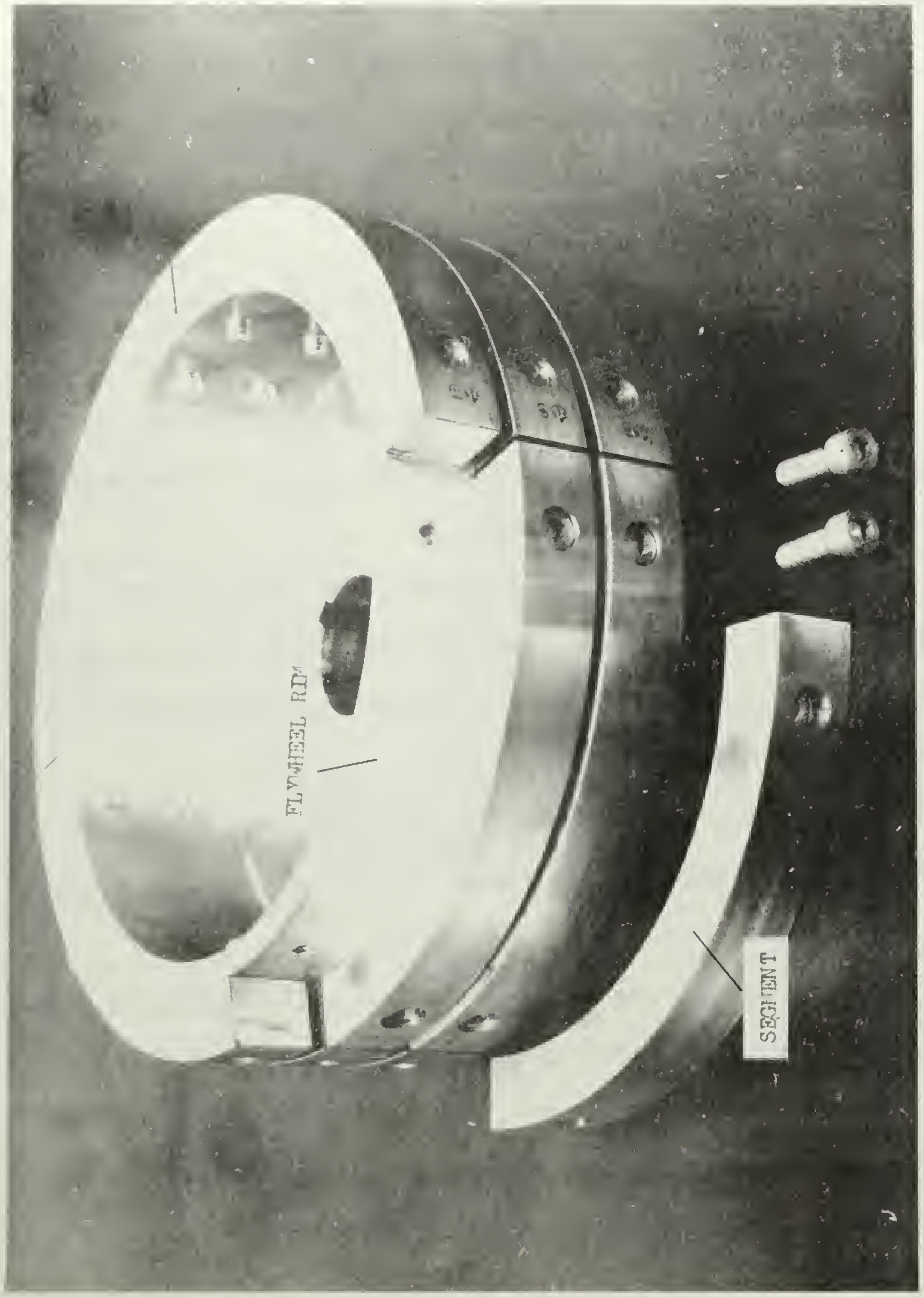


FIGURE 6. CHANGEABLE MASS FLYWHEEL

inertia. The mass moment of inertia of the rotating elements of the test system, with the exception of the armature of the main drive motor, may be changed by approximately 40% by the addition of all the segments to the bare rim. See table I. Since the masses of the individual segments vary somewhat, table II shows the mass effect of each segment.

In order to preserve balance, the segments must be added to the rim in pairs, each segment placed 180° from its mate on the rim.

The bearing hangers, figure 2, were modified from the original design to provide more shaft clearance, and felt oil seals were provided on both ends of each shaft.

An auxiliary base, figures 2,3, and 7, was made to raise the entire test unit 5 inches from the bed plate. This was necessary in order to match the input shaft of the test unit with the shaft of the main drive motor.

The test unit, integral with the auxiliary base, is bolted and pinned to the bed plate to insure correct alignment with the main drive motor.

The cover plates, figure 7, of the test unit, originally solid quarter-inch stainless steel plate, were modified to provide two view ports in the top cover plate, one for the drive section, and one for the test section. The operation may also be observed from the original end view port, as seen in figure 7.

MAIN DRIVE

The driving power for the test unit is a 5 KW, 250 volt D.C.

TABLE I

WEIGHTS, MASSES, AND MASS MOMENTS OF INERTIA OF ROTATING ELEMENTS OF THE TEST UNIT

ELEMENT	MASS (pounds)	MASS (slugs)	I_m (lb-ft-sec^2)
DRIVE GEAR	27.9	0.865	0.02835
DIVE LOAD GEAR	13.9	0.430	0.01410
TEST DRIVE GEAR	4.3	0.133	0.00401
TEST GEAR	2.88	0.0895	0.00271
HOLLOW SHAFT	19.4	0.603	0.00210
SOLID SHAFT	21.0	0.652	0.00227
LOADING RAM	4.65	0.145	0.00030
FLYWHEEL	8.40	0.261	0.01270
SEGMENTS	8.942	0.278	0.026437
	TOTAL I_m (without segments)		0.06654
	TOTAL I_m (with segments)		0.09297

$$\frac{0.09297}{0.06654} = 1.396$$

TABLE II

WEIGHTS, MASSES, AND MASS MOMENTS OF INERTIA OF FLYWHEEL SEGMENTS

RING	SEGMENT	WEIGHT (pounds)	MASS (slugs)	I_m (lb-ft-sec^2)
B	0	0.767	0.02384	0.002267
	1	0.758	0.02357	0.002242
	2	0.761	0.02363	0.002247
	3	0.765	0.02378	0.002262
C	0	0.7215	0.02243	0.002133
	1	0.724	0.02251	0.002141
	2	0.721	0.02241	0.002131
	3	0.720	0.00238	0.002129
D	0	0.7525	0.02339	0.002225
	1	0.750	0.02332	0.002218
	2	0.751	0.02335	0.002221
	3	0.751	0.02335	0.002221

The mass moment of inertia of the flywheel segments is taken about an axis through the center of the flywheel rim and parallel to the shaft on which it is mounted.

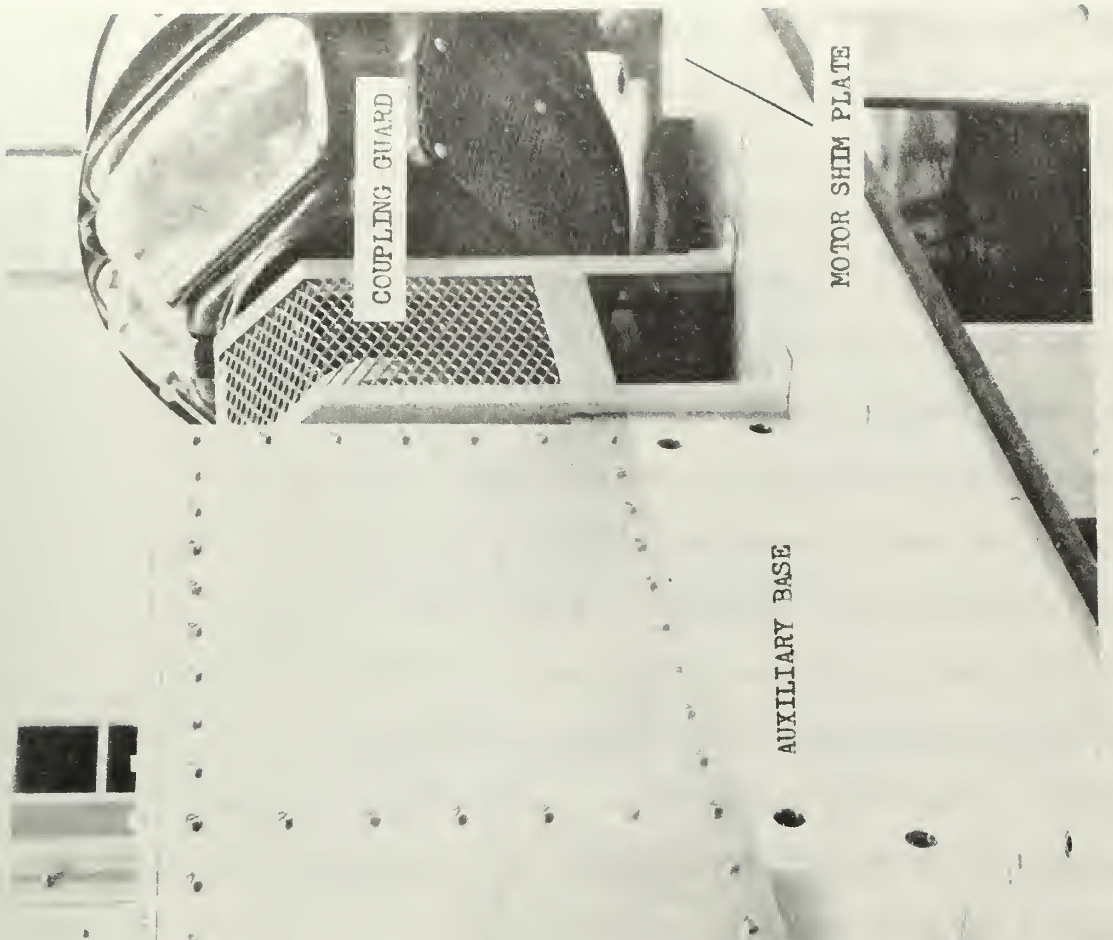


FIGURE 7. TEST UNIT SHOWING BASE AND COVER PLATES

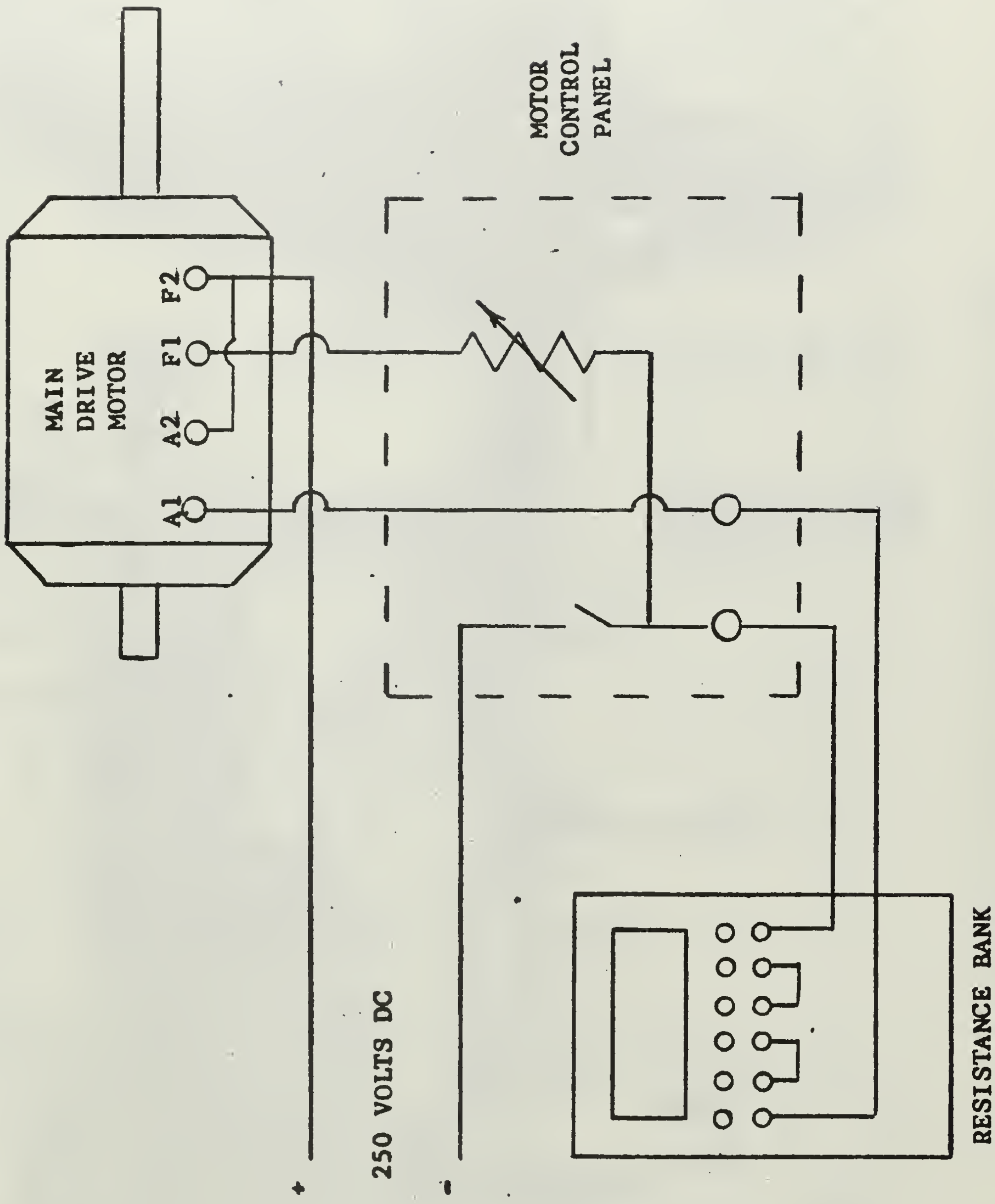
electric motor. A. D.C. motor was used because of the need for speed control over a large range. The motor is controlled by a switch and resistance bank, used for starting and speed control from 0-400 rpm, and a variable field resistance for speed control from 400-1200 rmp. A schematic for the main drive motor control circuit is shown in figure 8.

The motor drives the test unit through a flexible coupling and a shaft adapter, shown in figure 9. The flexible coupling employs a rubber "biscuit" to transmit power. This type of coupling was used to reduce any vibrations that might be transmitted from the motor armature to the test unit.

The motor is bolted and pinned to the bed plate, utilizing shim plates to align the motor shaft to the test unit shaft. See figure 7. Transverse alignment was accomplished by mounting a dial indicator on the shaft adapter flange, and indicating to the bearing hanger of the test unit.

LUBRICATION SYSTEM

The lubrication system consists of two essentially identical systems, one for the drive section, and one for the test section. Both systems are shown schematically in figures 10 and 11. Oil is drawn from a five gallon sump tank by a positive displacement pump. It then is fed through a tubing system to a feed valve, a bypass valve, and a pressure relief valve. The bypass valve provides for circulating the oil from the sump, through the pump, and back to the sump, in the event it is desired to heat the oil to a given temperature before using it as a lubricant. From the feed valve, the oil is fed through a filter, and then into lines feeding the



MAIN DRIVE MOTOR CONTROL CIRCUIT

FIGURE 8

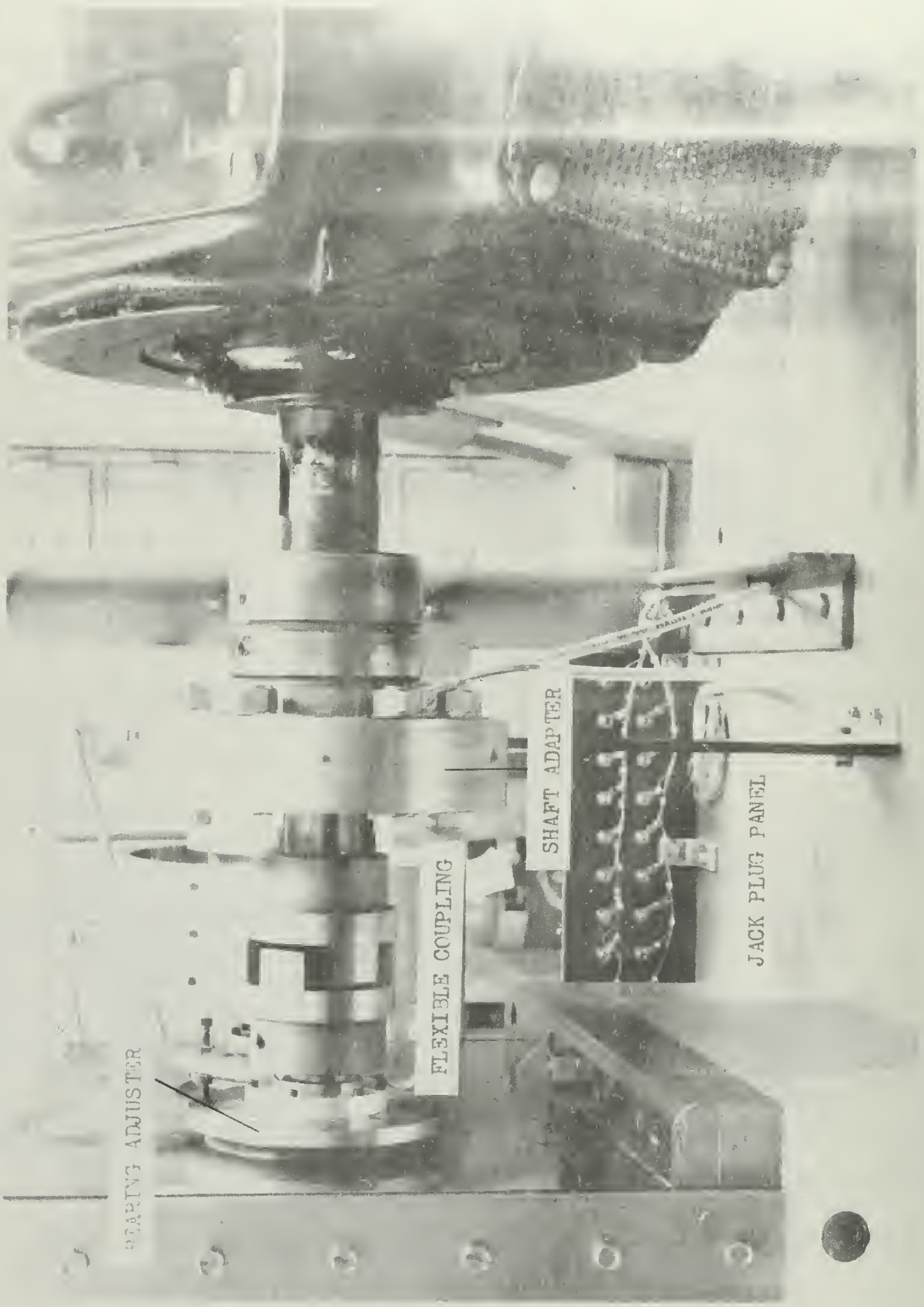
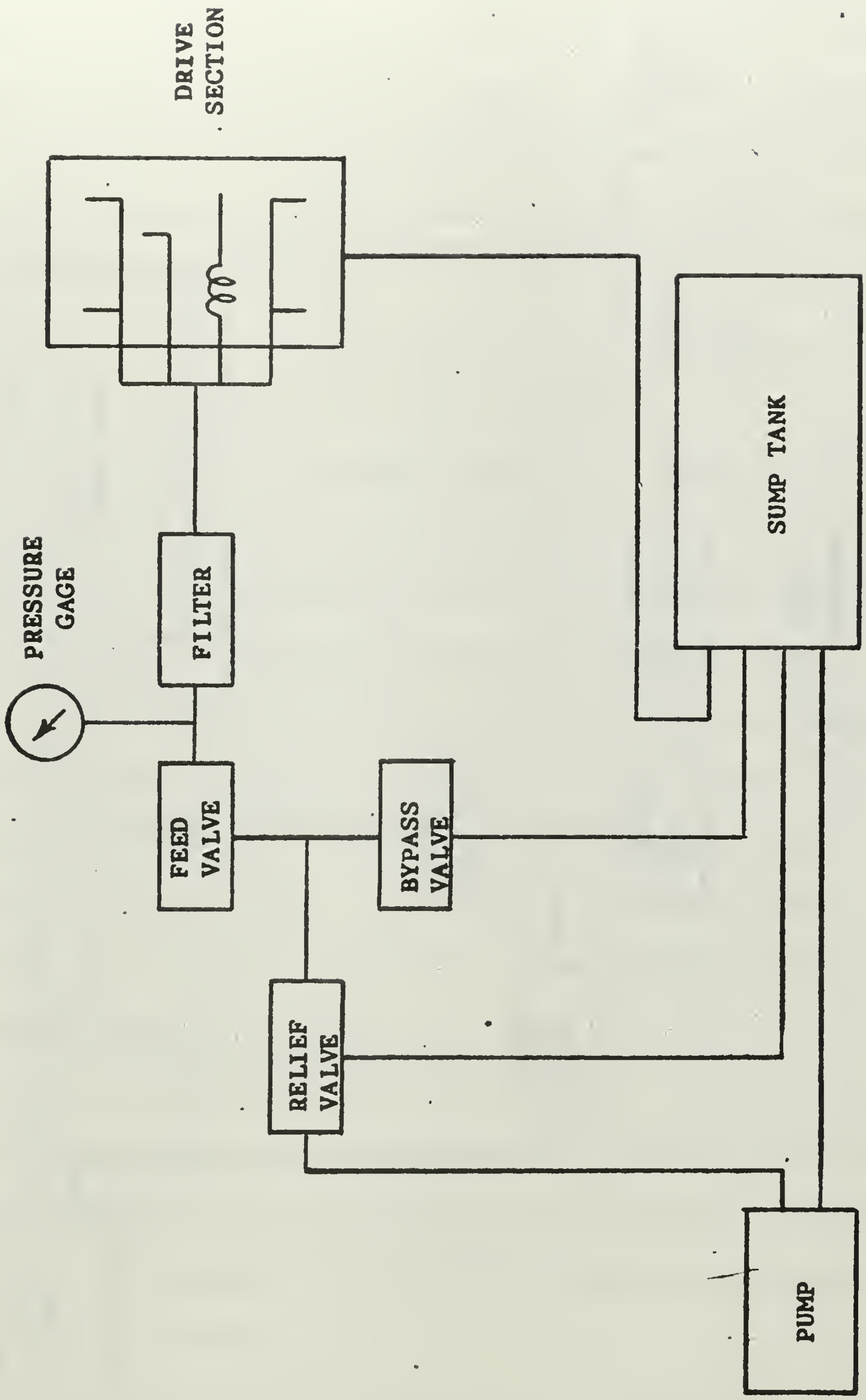
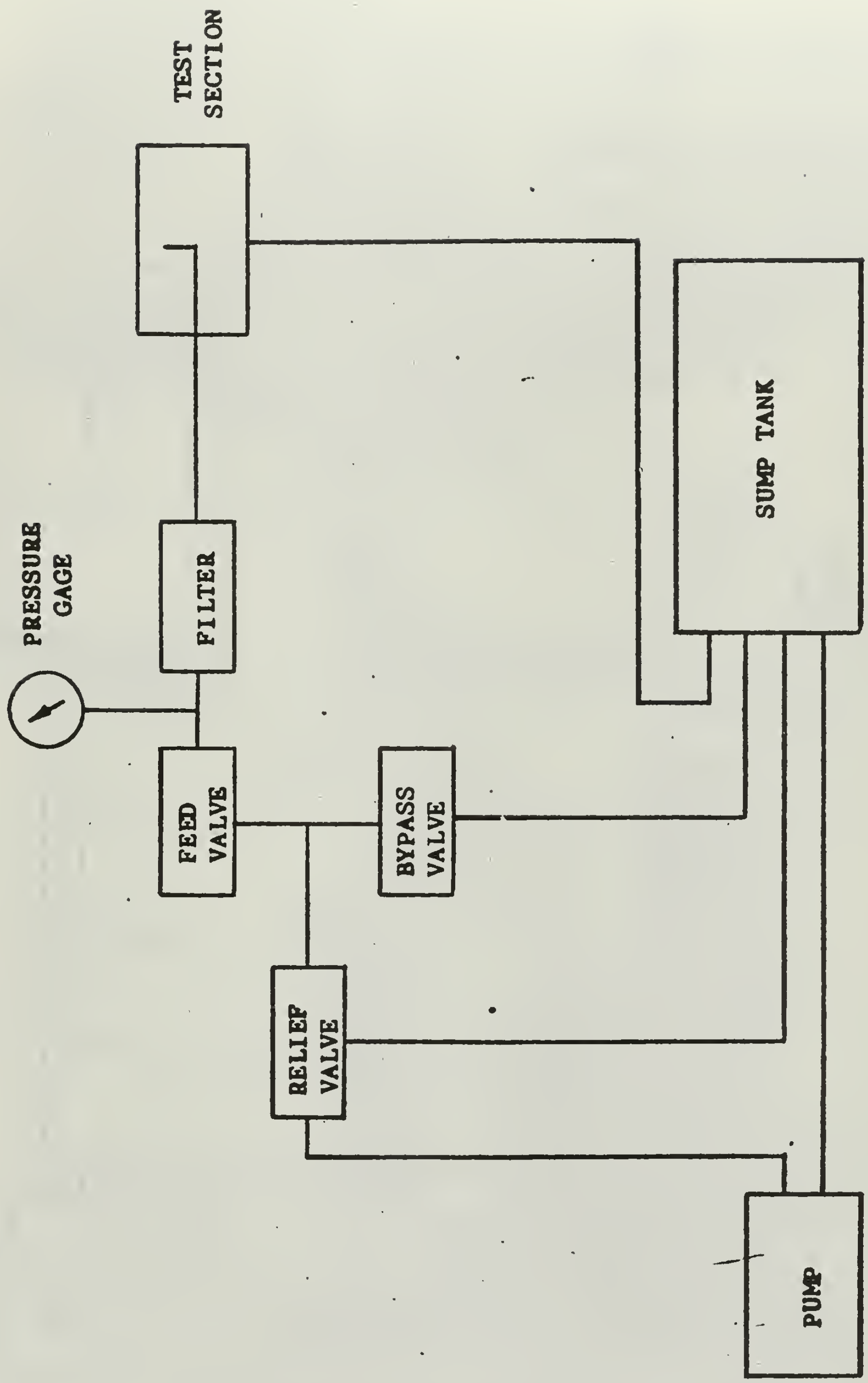


FIGURE 8. FLEXIBLE COUPLING AND SHAFT ADAPTER



SCHEMATIC OF DRIVE SECTION LUBRICATION SYSTEM

FIGURE 10



SCHEMATIC OF TEST SECTION LUBRICATION SYSTEM

FIGURE 11

various lubrication points in the test unit. A pressure gage is located in the lines just prior to the filters. This gives an indication of oil flow, and also an indication of filter condition. A dirty filter will show up as an unusually high gage pressure.

The drive section lubricant feeds the four shaft bearings through open nozzles, and the hydraulic loading ram thrust bearing through a closed line. This line is coiled as shown in figure 3 to permit axial movement of the ram assembly. Oil is sprayed directly on the helical drive gears just before they mesh.

The test section lubrication consists of an open nozzle that sprays the test gears just before they mesh.

Each lubrication system has its own water cooled heat exchanger, through which the oil flows before returning to the sump tank. The flow of cooling water to these heat exchangers is thermostatically controlled by the temperature of the lubricant in the sump tanks.

Both sump tanks are fitted with heating elements, again thermostatically controlled. This system permits the heating of lubricants to a specified temperature, and maintaining that temperature throughout any test runs.

The use of two separate lubrication systems, along with using felt oil seals on each end of each shaft, and gaskets between each section of the test unit, permits the use of different lubricants in each section, as well as maintaining each section's lubricant at a different temperature.

The feed valves for the two lubrication systems were intended to provide a means of regulating the flow of oil to the sections.

It was found in operation however, that the valves had to be wide open to provide adequate flow to the lubrication points.

All tubing in the lubrication systems is quarter-inch thin-wall stainless steel tubing, with high pressure steel fittings. It is felt that these fittings along with the high pressure needle valves used as feed valves, create too much of a pressure head in the system to permit the pump to furnish its rated flow of lubricant. For future use it is recommended that these fittings and valves be replaced with ones that restrict flow to a lesser extent. It is further recommended that the inlet lines from the sump tanks to the pumps be replaced with larger tubing than the present quarter-inch tubing. Experimentation with various inlet conditions to the pumps indicate that this is a large factor affecting flow rate through the pumps.

HYDRAULIC SYSTEM

The test unit is loaded by means of a hydraulic cylinder and ram assembly that displaces the drive loading helical gear with respect to the wide-face helical gear. The ram assembly and the two helical gears can be seen in figures 3, 4, and 12.

A schematic representation of the hydraulic system is shown in figure 13. Hydraulic fluid is taken from a three gallon sump tank by a positive displacement gear type pump, and sent through a filter to a feed needle valve, a bypass valve, and a pressure relief valve. From the feed valve, the fluid goes through a three way selector valve. The selector valve controls loading and unloading of the ram, and also has a bypass position. In the bypass mode, and pressure in the load or unload lines is held. Exhausting of this pressure is accomplished by placing the selector in the mode opposite to that which is to be

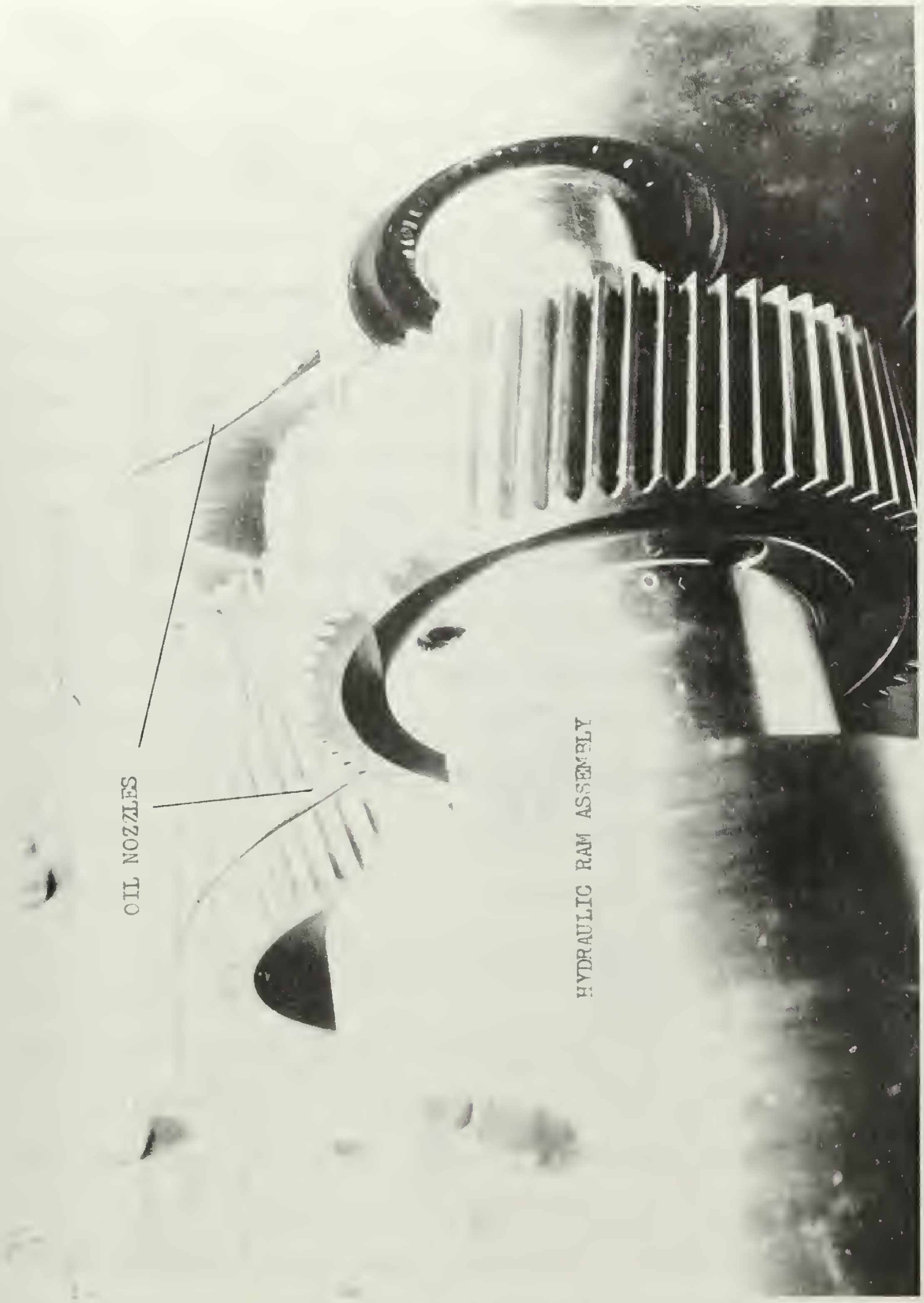
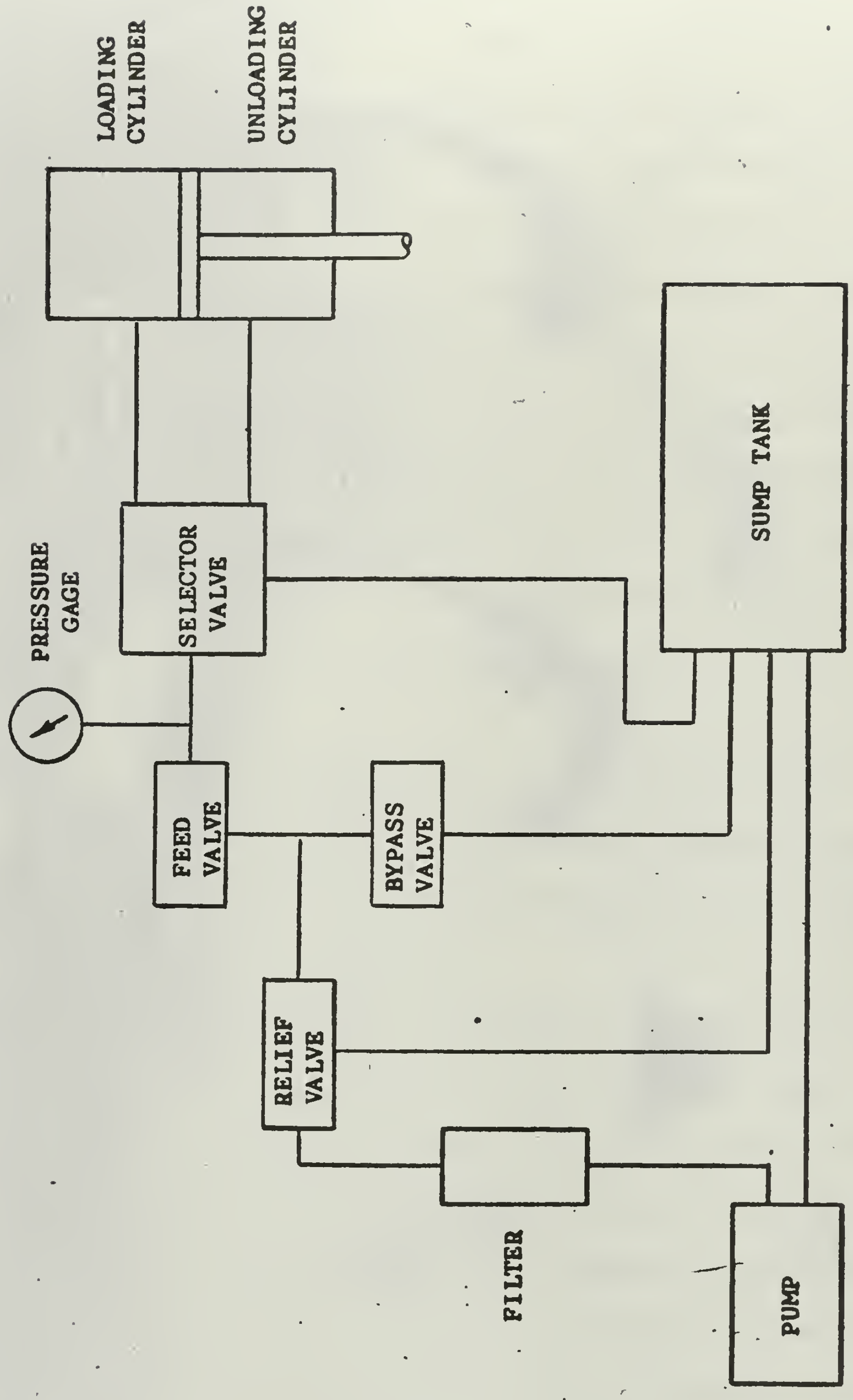


FIGURE 12. LOADING RAM AND GEAR



SCHEMATIC OF HYDRAULIC SYSTEM

FIGURE 13

exhausted. In other words, when loading, the unload cylinder is vented to exhaust, and vice versa. Any exhausted fluid is returned to the sump, as is all the fluid supplied to the selector valve when the valve is in the bypass position.

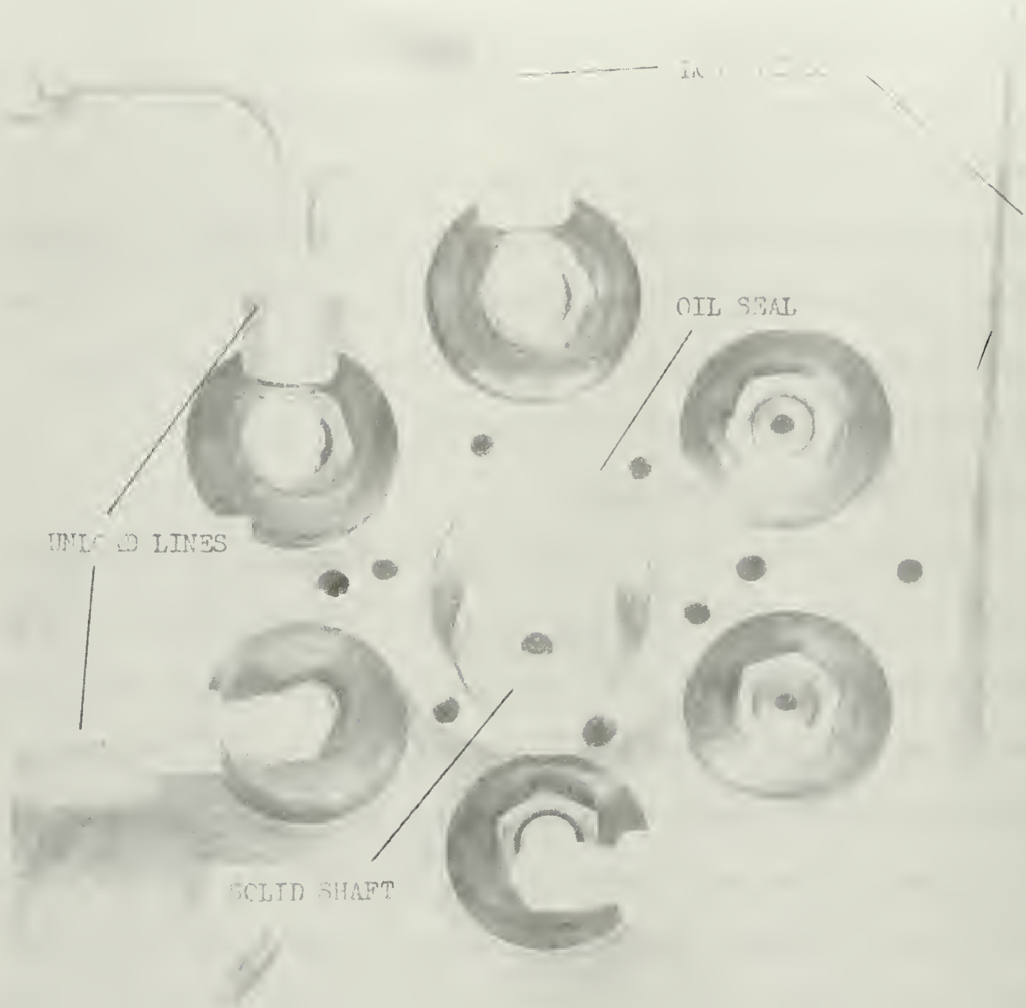
All hydraulic lines are quarter-inch thin-wall stainless tubing with high pressure fittings. The lines pass through the side cover plate of the test unit, into the test section, and are fitted into the ends of four hollow hydraulic ram support rods, as shown in figure 14. These rods pass through the bearing hanger that separates the test section from the drive section.

BASE AND SUPPORTING ASSEMBLIES

The main frame for the entire machine consists of two eighteen inch "I" beams, seven feet long, which are fastened together by short cross pieces welded in place. Quarter-inch steel plates are welded to the bottom of this assembly, which are in turn bolted down to two concrete blocks, with rubber insulating pads between the concrete and the steel frame. This is shown in figures 1 and 16. To the top surface of the two "I" beams, a one inch aluminum plate, known as the bed plate, is bolted. To the bed plate are bolted the test unit, with its auxiliary base, the main drive motor, motor coupling shield, lubrication filters, and all control panels.

The underside of the bed plate has bolted to it the water cooled heat exchangers, oil return lines, and electrical conduit.

Two lower platforms are formed by bolting one inch aluminum plates inside the space formed by the two "I" beams, at either end of the main frame. These two plates can be unbolted and slid out of the main frame for ease of maintenance of the equipment mounted



[illegible]

[illegible] and [illegible] hollow ram assembly support rods. The two [illegible] on the right are [illegible].

FIGURE 15. TEST UNIT

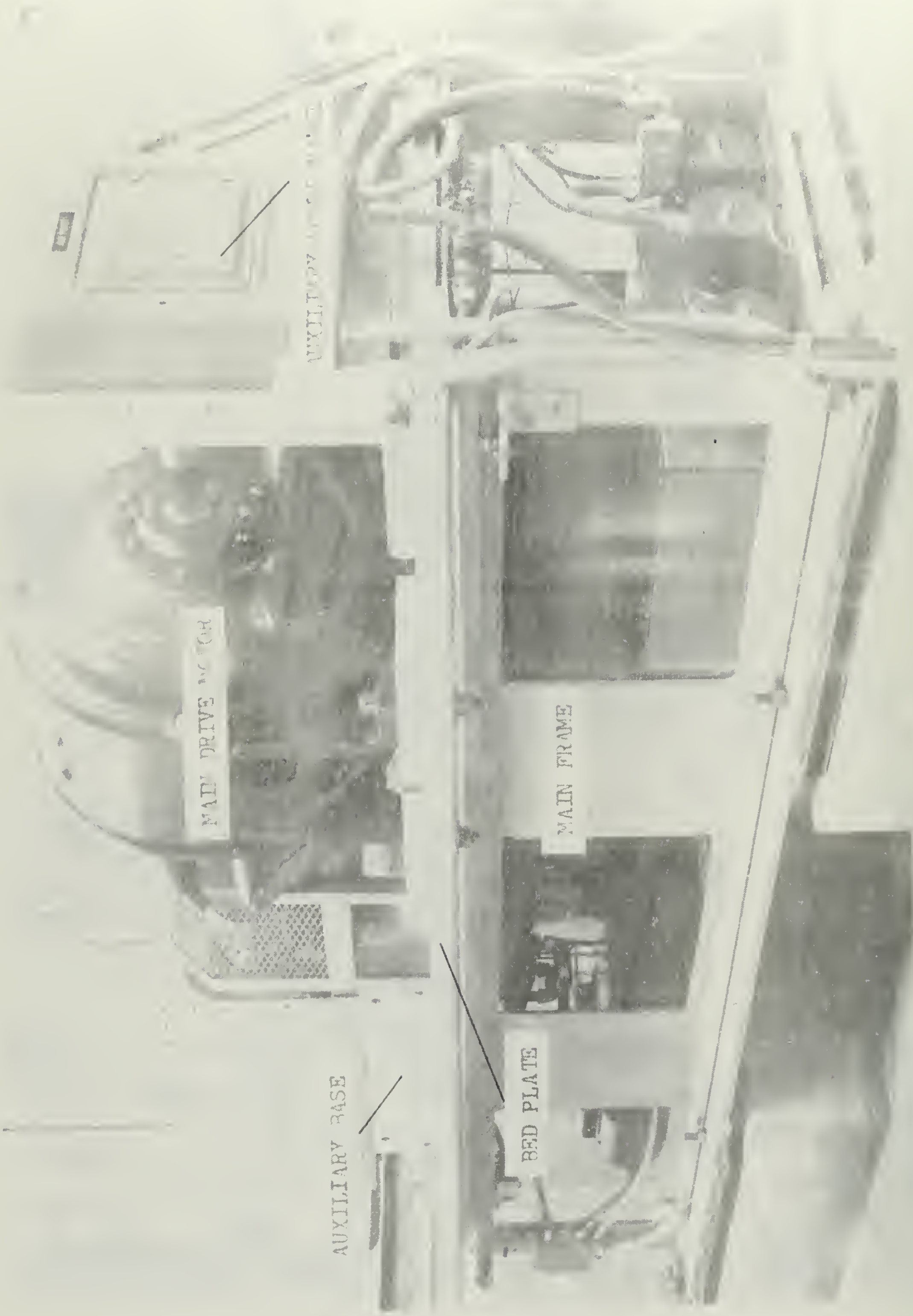


FIGURE 16. TEST MACHINE [illegible]

on them. This equipment, figures 17 and 18, consists of the two lubrication sump tanks, the hydraulic fluid sump tank, all lubrication and hydraulic pumps, and the hydraulic filter.

Control panels consist of three-eighths aluminum plates, mounted on one-inch angle-iron brackets, which are in turn bolted to the bed plate. There are four control panels in all.

The main control panel, shown in figure 19, holds the two lubricant pressure gages, the hydraulic pressure gage, all feed and bypass valves for the lubrication and hydraulic systems, and the load selector valve for loading and unloading the test unit hydraulically.

The main drive motor panel, shown in figure 20, contains a power switch, a variable resistance for the motor field circuit, and two jack plugs used to connect the starting resistance bank into the armature circuit of the motor.

An auxiliary power panel, figure 16, houses switches and fuses for all pumps, the heating elements for the lubrication sump tanks, and the water solenoid valves and their thermostatic controls for the cooling water to the heat exchangers.

The heater and thermostat panel, shown in figure 21, holds two variable heater controls, one for each of the lubrication sump tanks, and the thermostatic controls for the cooling water.

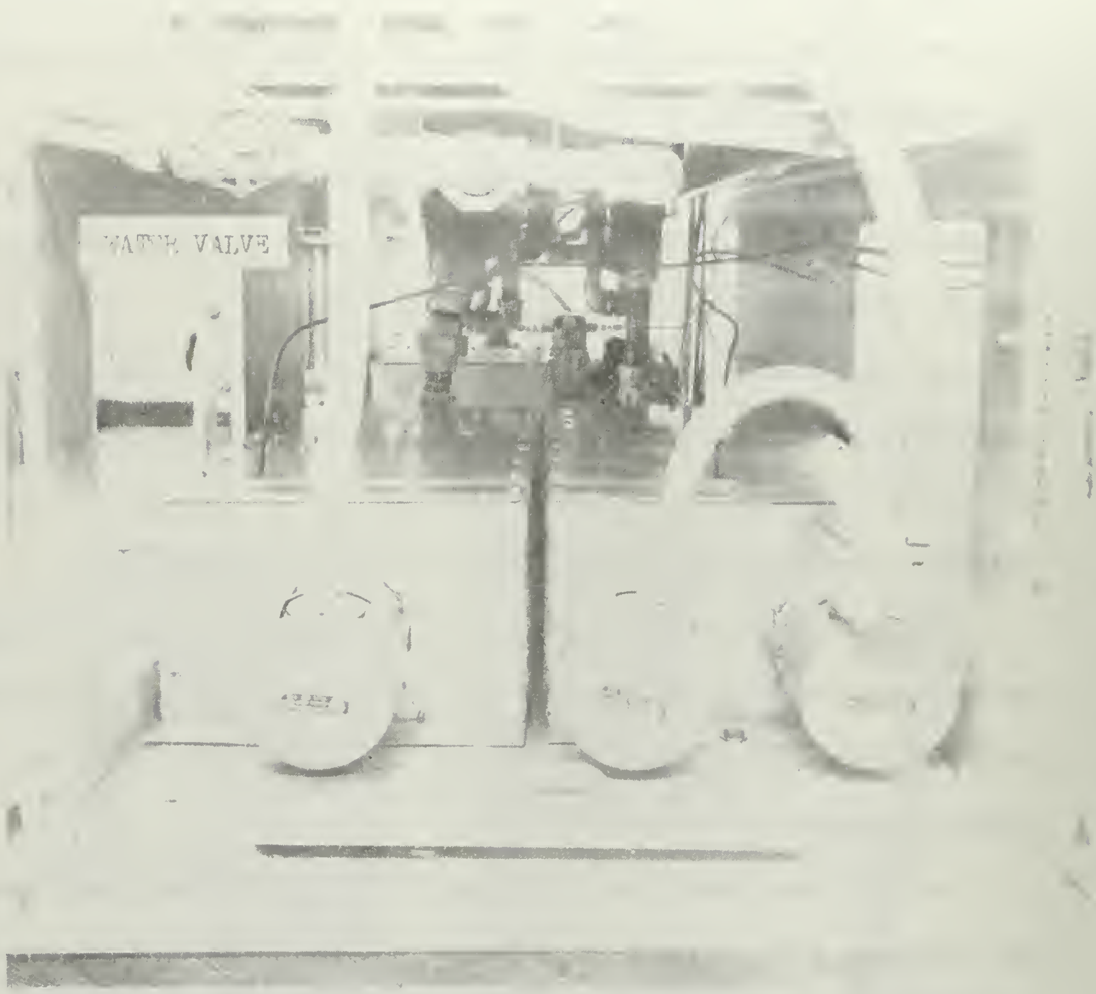


FIGURE 17

lower platform, showing the two lubricant sump tanks with their heaters. The cooling water supply valve is seen in the upper part of the figure.

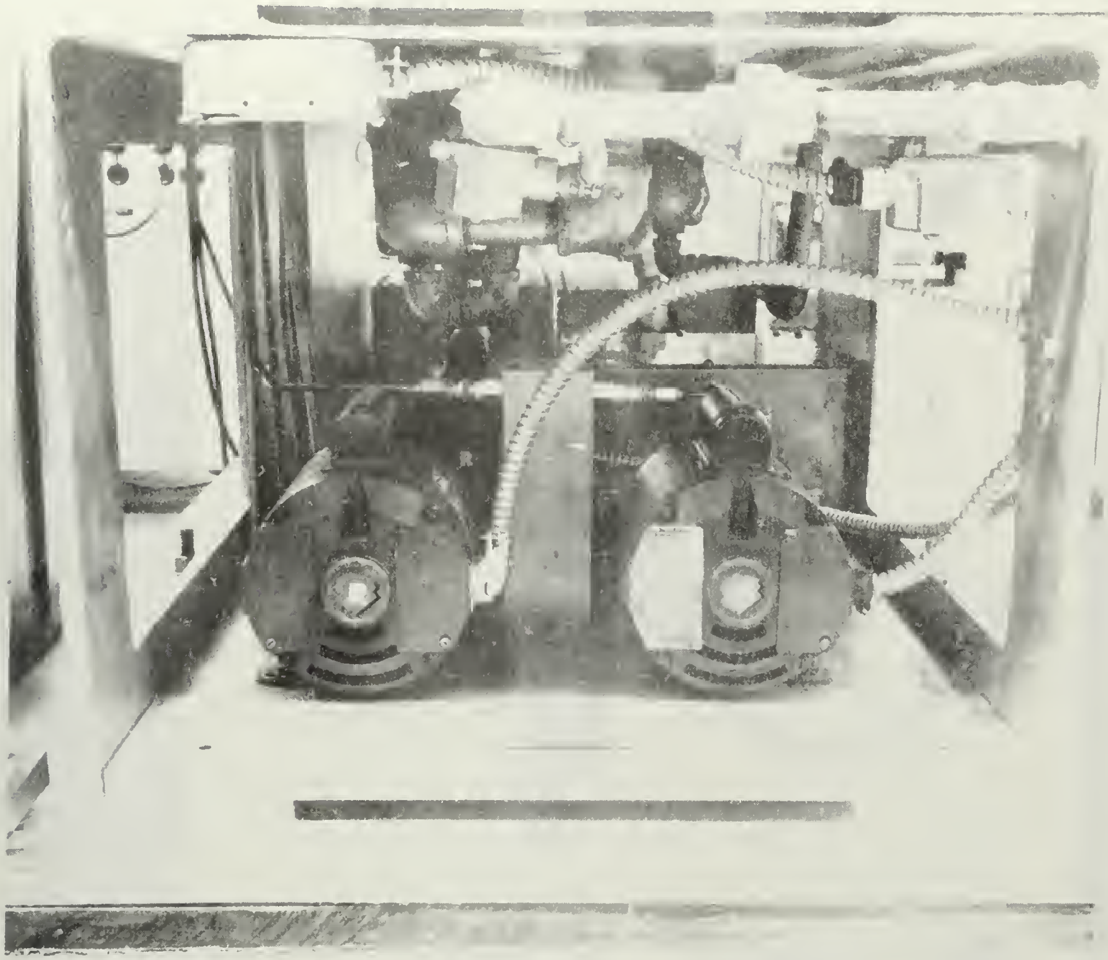


FIGURE 18

Lower platform, showing the hydraulic pump and sump tank, and one of the lubricant pumps. Some of the oil return piping and cooling water piping may also be seen.

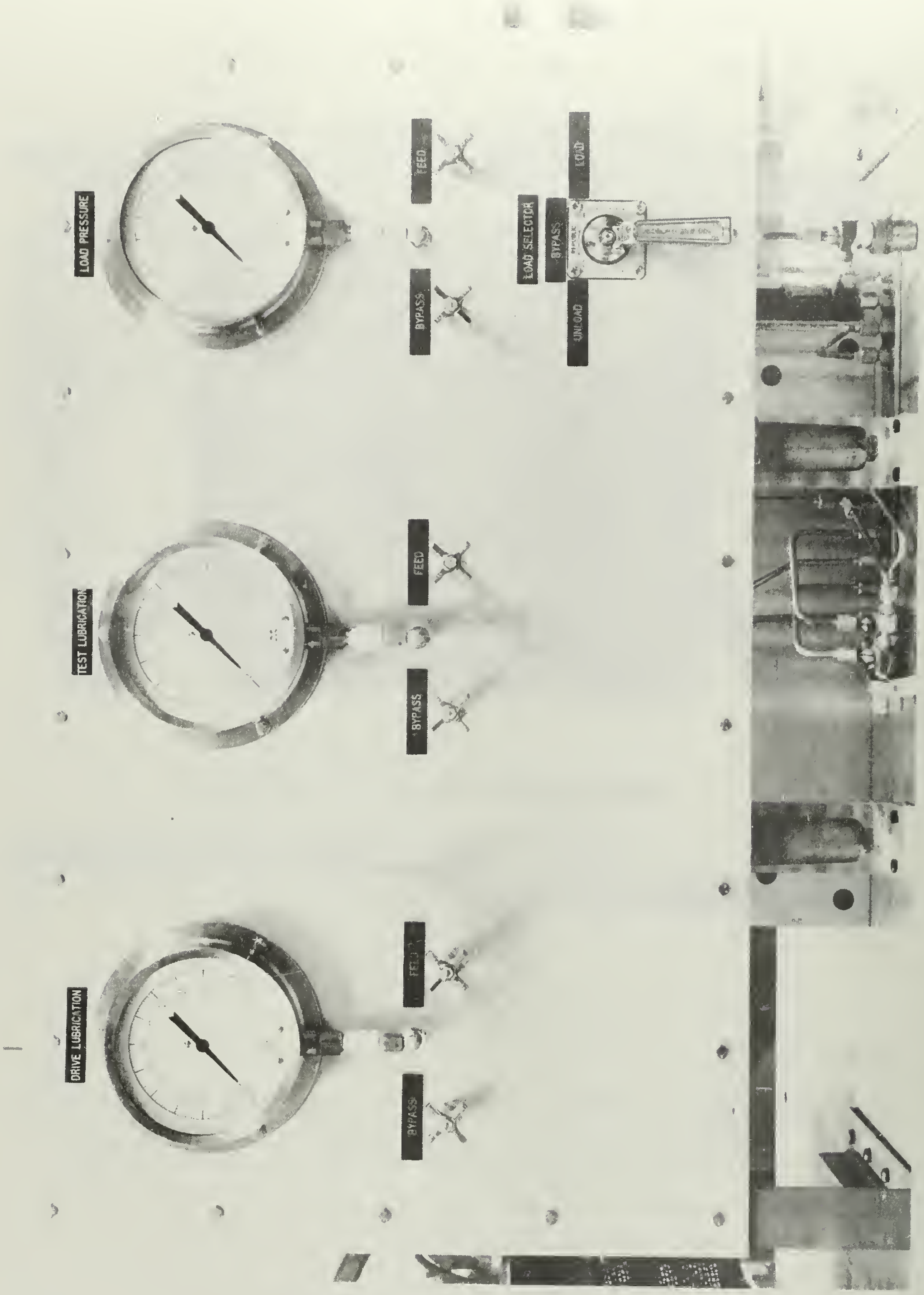


FIGURE 19. MAIN CONTROL PANEL

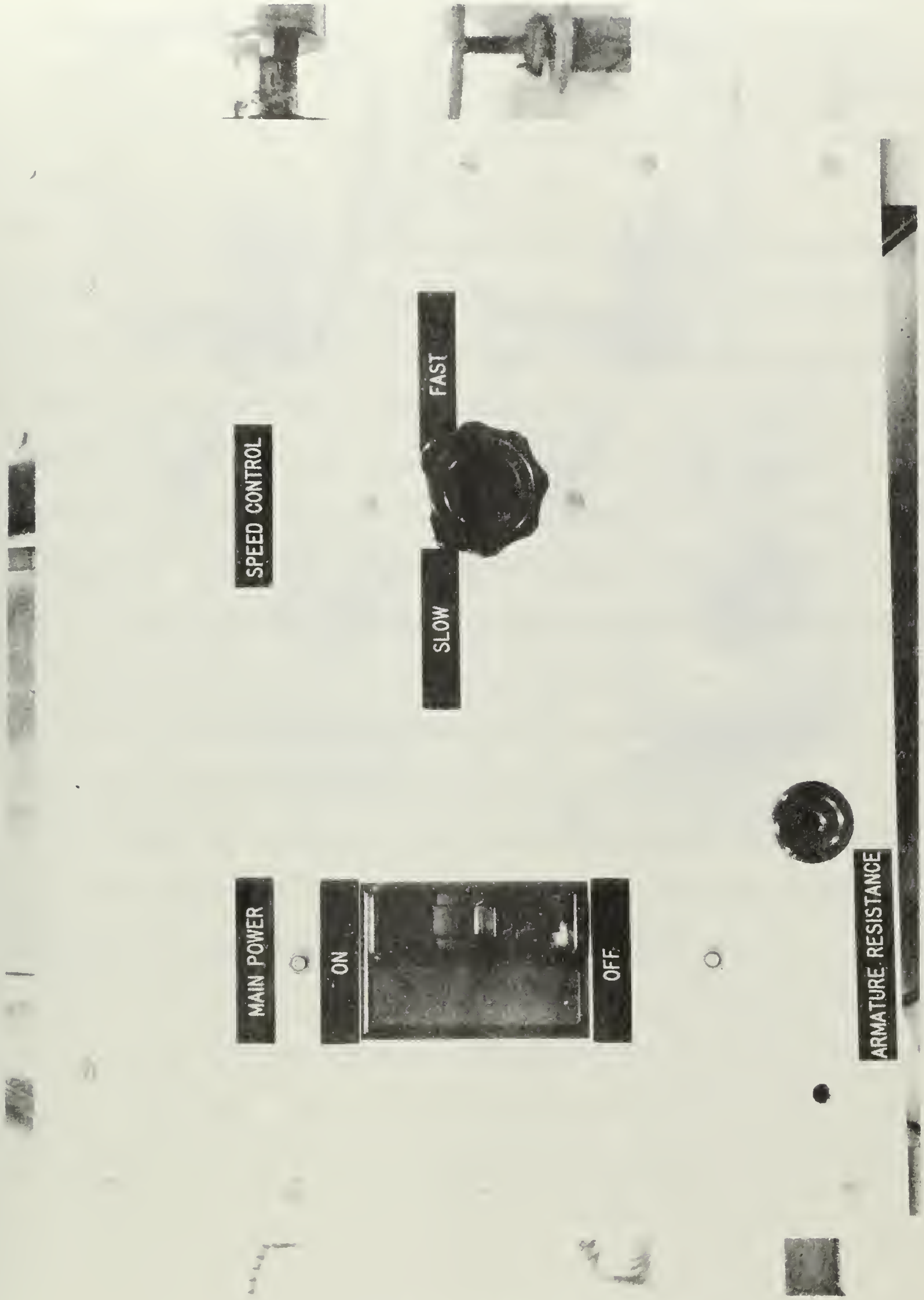


FIGURE 20. MAIN DRIVE MOTOR CONTROL PANEL

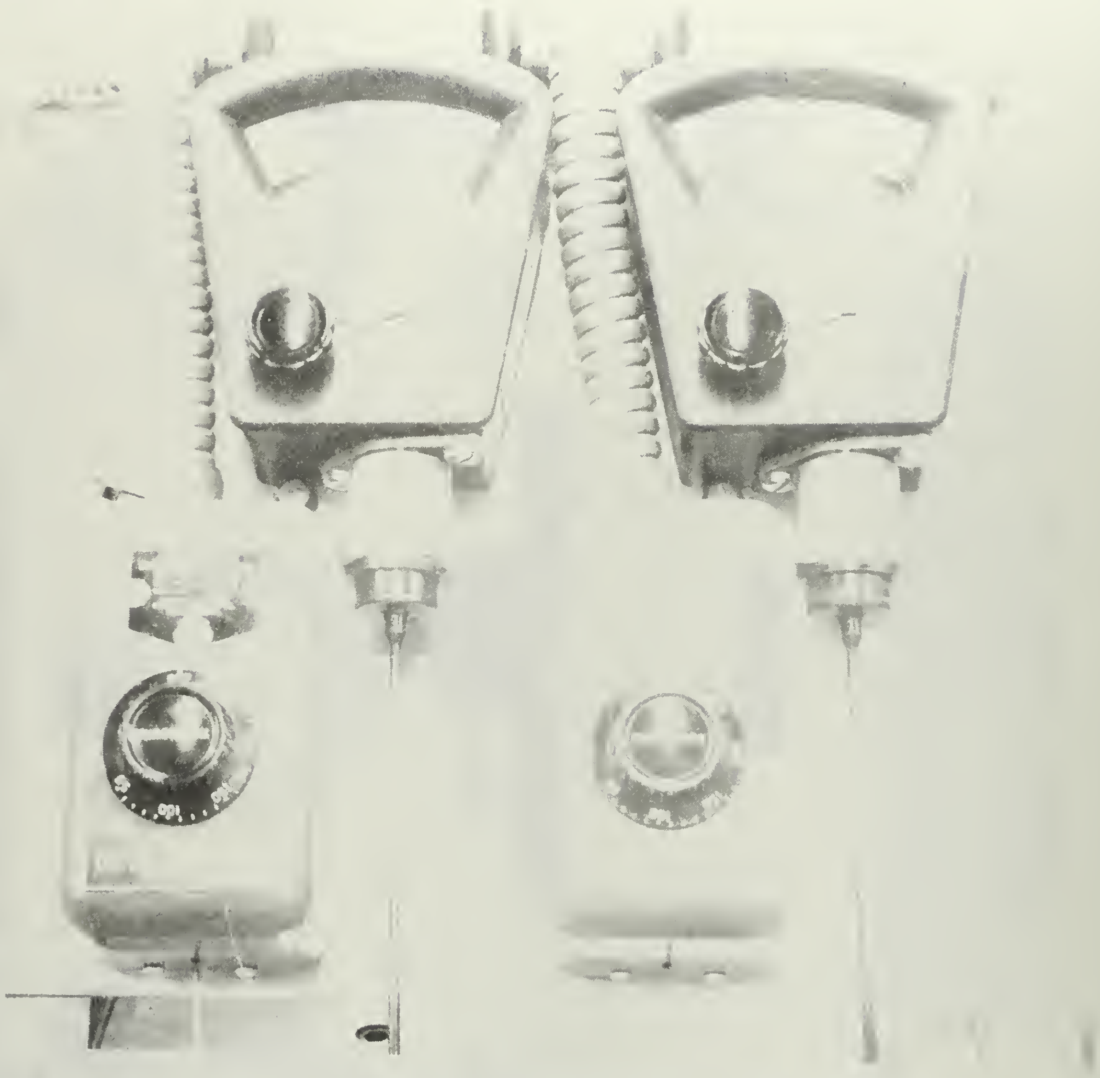


FIGURE 21. HEATER AND THERMOSTAT PANEL

III. OPERATION

Upon completion of the construction of the testing machine, it was run, with a light load, for an actual operation period of fourteen hours, varying the speed from about 50 rpm up to 1500 rpm. This was done prior to any instrumentation in order to let parts wear in, and to check bearing adjustment.

Bearing temperatures were continually noted during the running-in phase. If a bearing showed a tendency to heat up, it indicated that thermal expansion of the shafts had caused all bearing end play to be taken up, and the bearing was running with too much preload. This was verified by stopping the machine and turning it over by hand. If the unit was hard to turn over, the bearing adjusters were backed off in .0005 inch increments, the machine allowed to cool, and then run again at the same speed at which the heating had occured for a minimum of one hour. If no abnormal heating occured after the above adjustment had been made, the speed was increased by several hundred rpm, with the same check on temperature rise run again.

During the running-in phase, and the subsequent testing phase, an operating procedure was evolved, which is presented in the following pages in the form of operating instructions.

START UP

1. Start 250 volt D.C. generator and turn on both D.C. and A.C. power switches on the main power panels, located in room 101, building 234.
2. Close all lubrication and hydraulic feed valves. Open all lubrication and hydraulic bypass valves. Place load selector valve in the bypass position.
3. Turn on lubrication and hydraulic pump switches located on auxiliary power panel.
4. If the lubricant is to be heated to a test temperature, turn on power to oil heaters and water solenoid valves. Open water supply valve. Set heater controls and thermostats to the desired temperatures. If lubricant is not to be heated, this step may be omitted.
5. When lubricant is at the desired temperature, open lubrication feed valves and close lubrication bypass valves. Check pressure gages for indication of proper lubricant flow. Drive system pressure should be 15 - 30 psig, and test system pressure 10 - 20 psig. Pressures may read higher than these values for a minute or so when the feed valves are first opened, but should drop to the indicated values. A constant reading higher than that specified indicates a dirty filter. As an additional check, lubricant flow may be observed through the view ports in the top cover plate of the test unit.
6. Insure all resistance bank switches are off, or in the down position, placing maximum resistance in the main drive motor

armature circuit. Turn speed control knob to maximum counter-clockwise position (slow).

7. Turn on main power switch, and control test unit speed with resistance bank switches from 0 - 400 rpm. Above 400 rpm, speed is controlled by the speed control knob. Under no circumstances should the speed control knob be used before all resistance bank switches are on (up). For high speeds and high loading of the test unit, it may be necessary to short out the resistance bank with a short lead.

LOADING

1. With the machine running at any speed, open the hydraulic feed valve. Place the load selector valve in the load position until the desired loading is obtained. Once the desired loading is obtained, it may be held by placing the load selector in the bypass position.

   NOTE: For most loading, the feed and bypass valves are both open. This is to prevent too rapid a loading. For high loads it may be necessary to close the bypass valve somewhat until the load is obtained.

UNLOADING

1. Unloading is accomplished by placing the load selector valve in the unload position, with feed and bypass valves set as in loading. Once the load is removed from the test unit, place the selector valve in the bypass position. Failure to do this will result in loading the test unit in the opposite axial direction.

SHUTDOWN

1. Unload the test unit
2. Slow the machine with speed control knob. When speed knob is in its maximum counter-clockwise position, (about 400 RPM) the main power switch may be opened.
3. Turn off lubricant heaters.
4. Close cooling water supply valve.
5. Turn off power to water solenoid valves.

   NOTE: Step 4 must be accomplished before step 5 in order to avoid building up too much pressure in the cooling water system.
6. Turn off lubrication and hydraulic pumps.
7. Open A.C. and D.C. power switches at the power panels in room 101. Turn off D.C. generator.

IV. EXPERIMENTAL PHASE

GENERAL DISCUSSION

While the testing machine is designed to be used for a multitude of pursuits involving gears and lubrication, its immediate purpose it to investigate the loads on gear teeth.

The loads that come on the individual teeth of gearing can be many times greater than the transmitted load, which is a function of the power put into a geared device. These high tooth loads, called dynamic loads, are composed of the transmitted load and a load increment due to dynamic forces. These dynamic forces are caused by angular accelerations and decelerations of the gear masses, and any attached masses.

The angular accelerations and decelerations are related to a function called tooth error. Tooth error is made up of several parts, profile error, indexing error, and radial runout being the more important ones.

Profile error is a result of gear cutting equipment being unable to generate a true involute profile. Indexing error is produced by inherent inaccuracies in indexing devices used to space the teeth on a gear blank. Radial runout can be caused by machining inaccuracies.

The total of these errors can be measured, and depending on the quality of the gear, can range from .00015 inch to more than .003 inch. Along with this tooth error, the mating of gear teeth is accompanied by tooth deflections and shaft deflections, which also affect the loading of the teeth.

During the mating of a pair of gear teeth, the error, which can be thought of as a high spot on a gear tooth, will cause the driving gear to slow down, and the driven gear to speed up, tending to cause the gear teeth to separate. The system load and applied power act to resist this action of the gear teeth, and bring them together again. [1] Buckingham states that this results in two pressure pulses on a gear tooth. One when the transmitted load is first taken up by the pair of mating teeth, and the second when the teeth, after bouncing apart, come together with an impact. It is this impact load that is the dynamic load.

Buckingham has predicted what these loads will be from experiment. [1] Since the dynamic loads are a function of gear masses, attached masses, shaft torsional stiffness, tooth error, and speed, the equations that Buckingham formulated are rather complex. He did develop a simplified "average" equation, that can be used with a fair amount of accuracy on systems of average masses.

It may be shown that the attached masses of the gear testing machine under consideration have little or no effect on the dynamic load acting on the test gears, according to Buckingham's analysis. Buckingham states that shafting less than two inches in diameter is flexible enough so that it does not transmit angular accelerations of the gears to the attached masses, so that dynamic impact loads are due only to the masses of the gears themselves. The shafts of the test unit are just two inches in diameter.

Dynamic load to transmitted load ratio verses speed curves for both Buckingham's exact analysis and his average equation were computed, and are shown in figure 22.

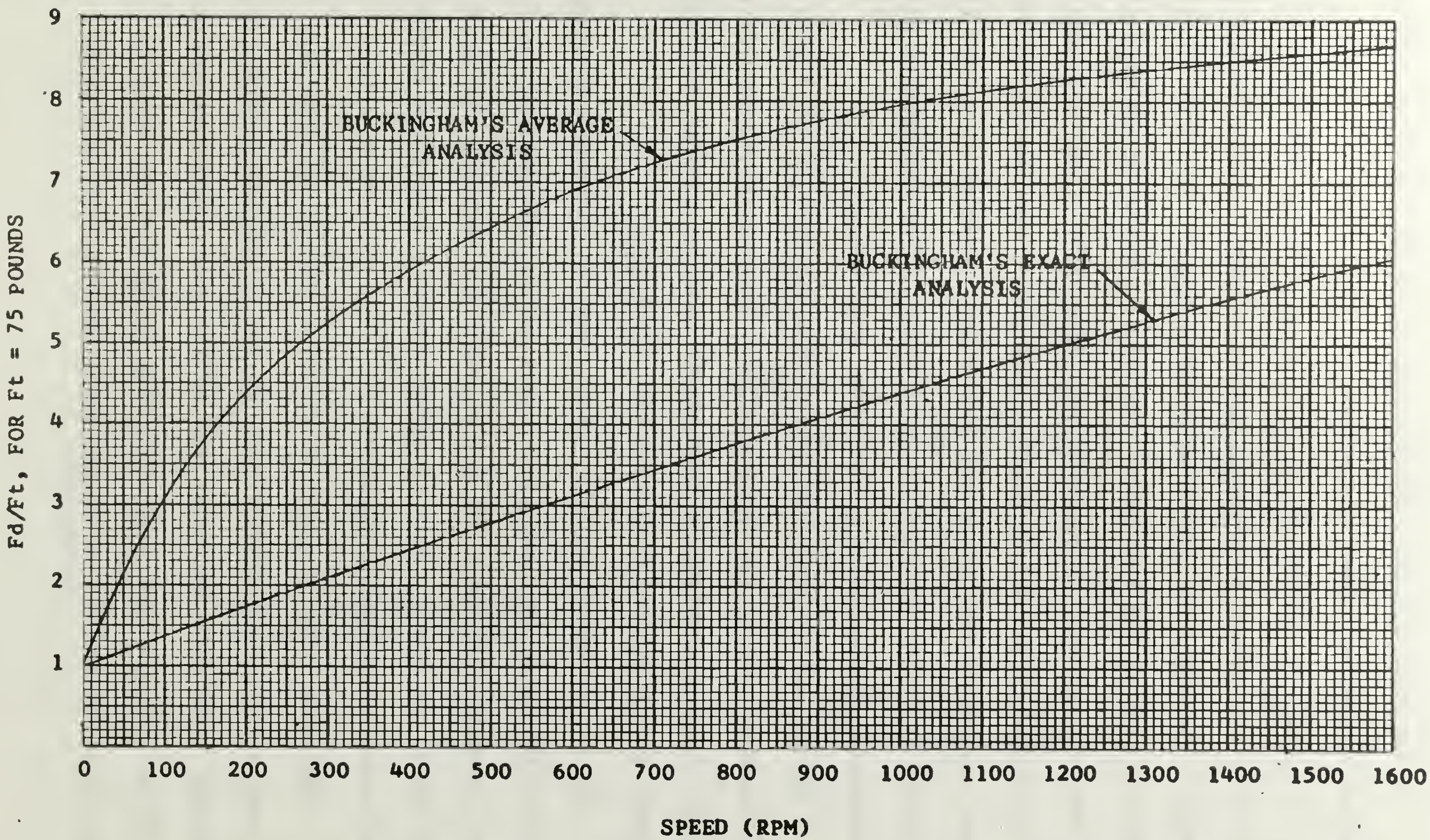


FIGURE 22, SHOWING DYNAMIC LOADING CURVES OBTAINED FROM BUCKINGHAM'S AVERAGE ANALYSIS AND HIS EXACT ANALYSIS.

INSTRUMENTATION

It was with dynamic load measurement in mind that the testing machine was designed and constructed.

In order to observe the loading of gear teeth, one tooth of the test gear was fitted with strain gages, arranged to form a four gage bridge. The gages are placed as shown in figure 23. This illustrates only one side of the test gear. The gages mounted on the opposite side of the gear are exactly opposite the gages shown. The four gage bridge is comprised of two active gages on the gear tooth itself, and two dummy gages mounted on the rim of the gear blank, oriented so as not to be affected by loads coming on the gear teeth. This arrangement gives temperature compensation to the strain gage bridge.

The two active gages are placed near the root of one tooth, on the unloaded side of the tooth profile. This position takes advantage of the stress concentration that exists near the root of the tooth to produce a high signal level from the strain gage.

Due to the direction of rotation, and the direction of loading, in the test unit, the load first comes on the instrumented tooth at the root opposite that where the active gages are mounted, and moves toward the tip of the tooth. This in effect causes a longer and longer lever arm to act on the strain gage.

The leads from the gages on the back side of the test gear are led through a small hole located near the hub of the gear, where they join the gage leads from the front side of the gear. From this point the leads go to a pin type connecter, held in an adapter, which in turn is mounted on the end of the hollow shaft carrying the test gear.

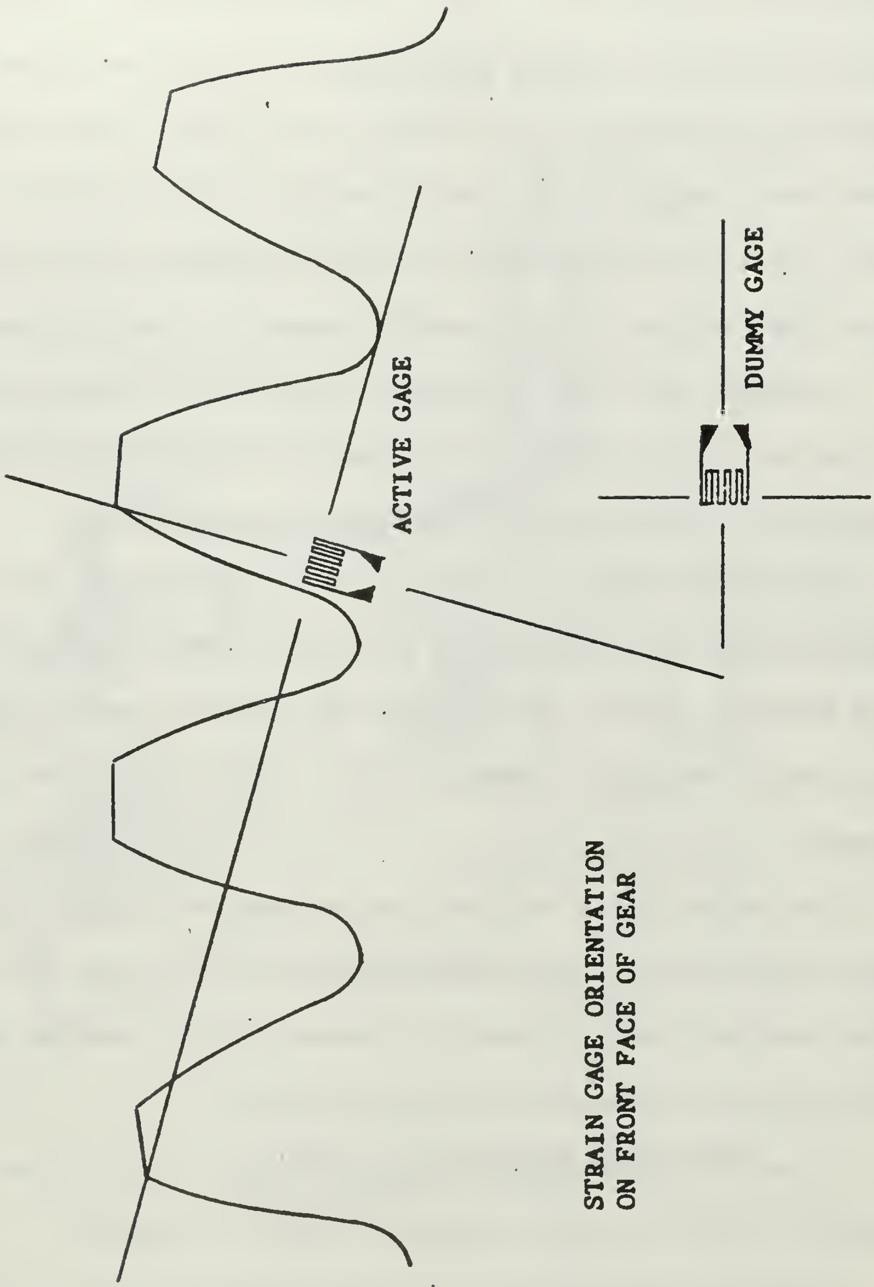

ACTIVE GAGE
DUMMY GAGE
STRAIN GAGE ORIENTATION
ON FRONT FACE OF GEAR

FIGURE 23

See figure 24. Leads from the adapter pass through the hollow shaft.

Transmitted load is measured by a strain gage bridge placed on the outside diameter of the hollow shaft. This is also a four gage bridge, set up to measure torque [7], which is directly proportional to the transmitted load. This strain gage bridge is located in the drive section of the test unit, and the leads from it pass through radial holes leading to the center of the shaft. See figure 25. Here these leads join the leads from the adapter serving the test gear, and all leads then pass through the shaft to the end that carries slip rings mounted on a detachable adapter, shown in figure 26.

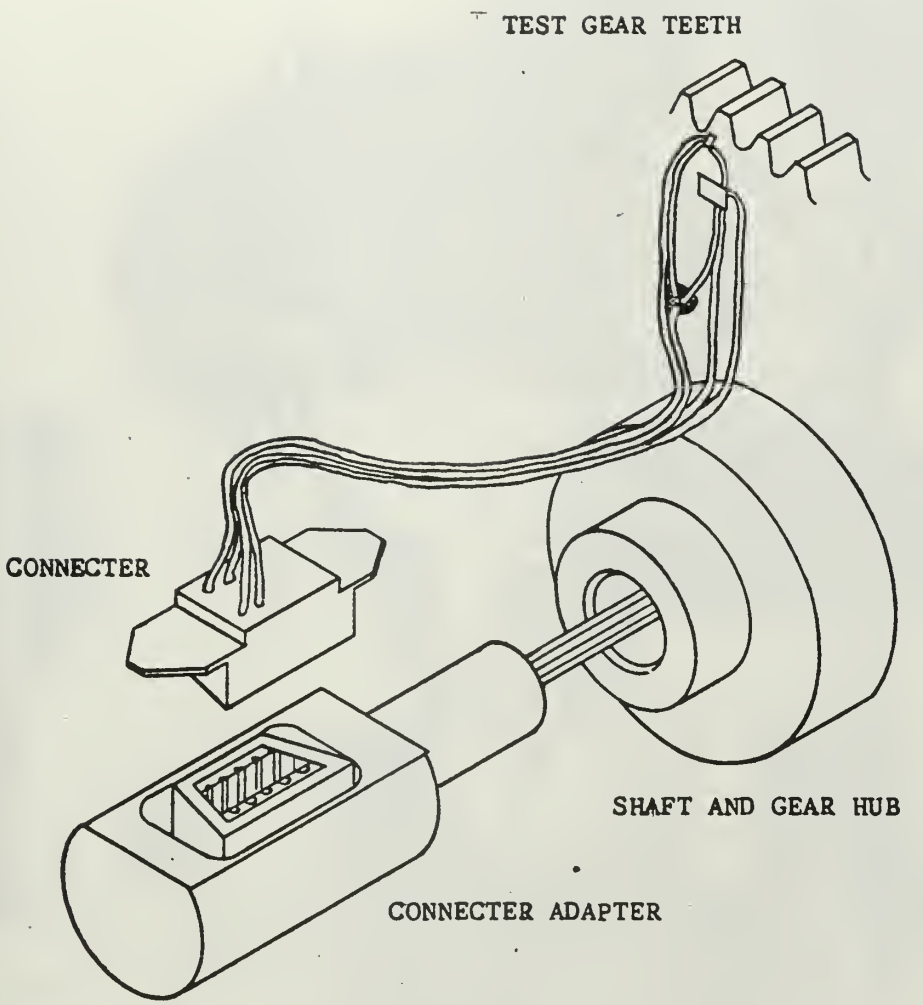
Complete four gage bridges are used on the instrumented portions of the test unit to eliminate as much as possible the effect of contact resistance of the slip ring brushes.

All strain gages, on both the test gear and the shaft, and all exposed leads are covered with a silicone rubber compound to protect them from the effects of hot oil. The rubber compound also constrains the leads, preventing movement of the leads from generating spurious signals.

The hollow shaft was packed with shredded sponge rubber and the ends sealed with silicone rubber after all the leads were in place. This again was done to prevent movement of the leads in the center of the shaft from generating spurious signals.

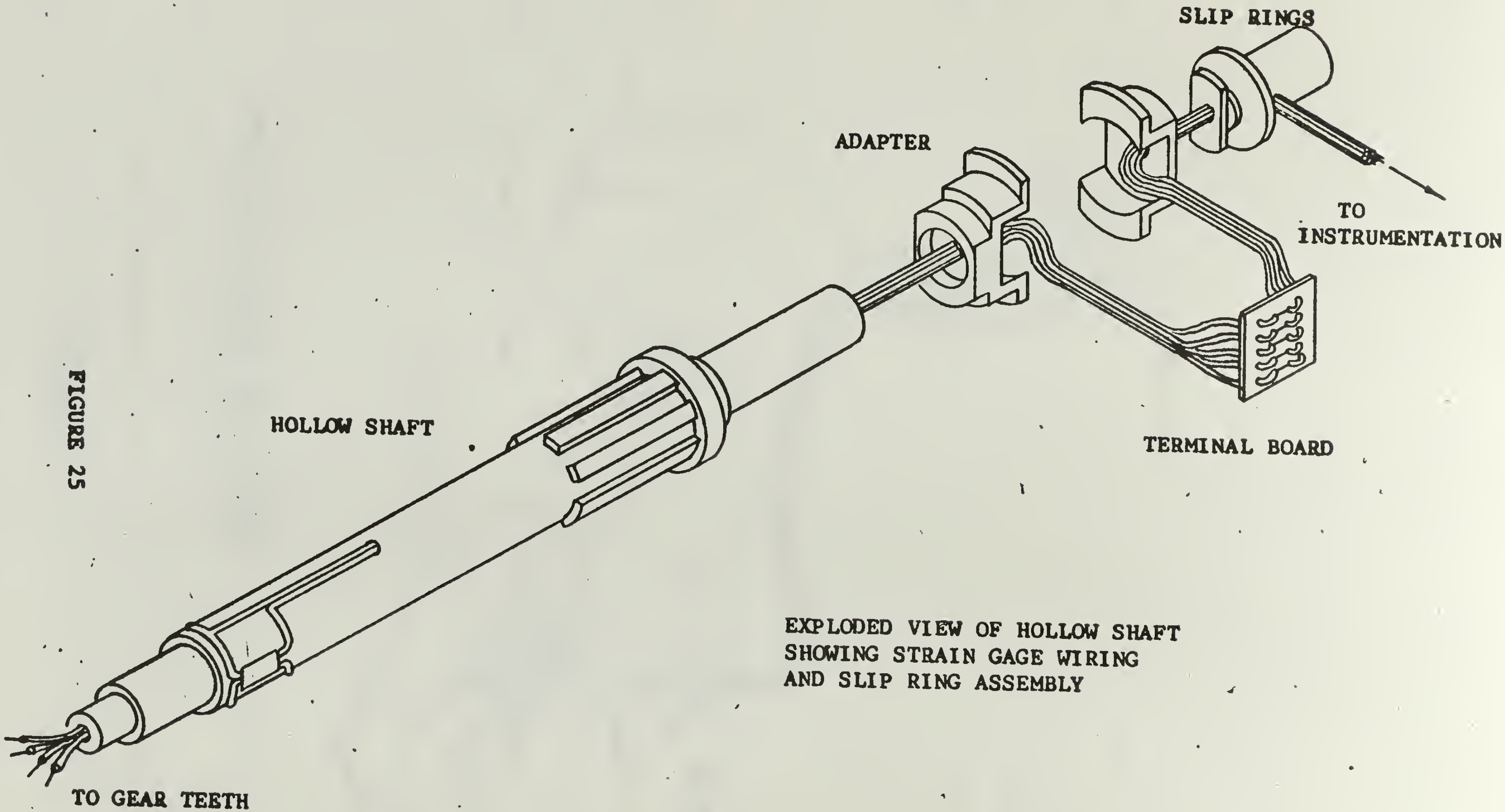
From the slip ring assembly, leads are run to a jack plug panel mounted on the bed plate, shown in figures 9 and 28.

External instrumentation consists essentially of strain gage recorders and a method of measuring the speed of the test unit, figures 27 and 28.



EXPLODED VIEW OF TEST GEAR WIRING
AND CONNECTER ADAPTER

FIGURE 24



EXPLODED VIEW OF HOLLOW SHAFT
SHOWING STRAIN GAGE WIRING
AND SLIP RING ASSEMBLY

FIGURE 25

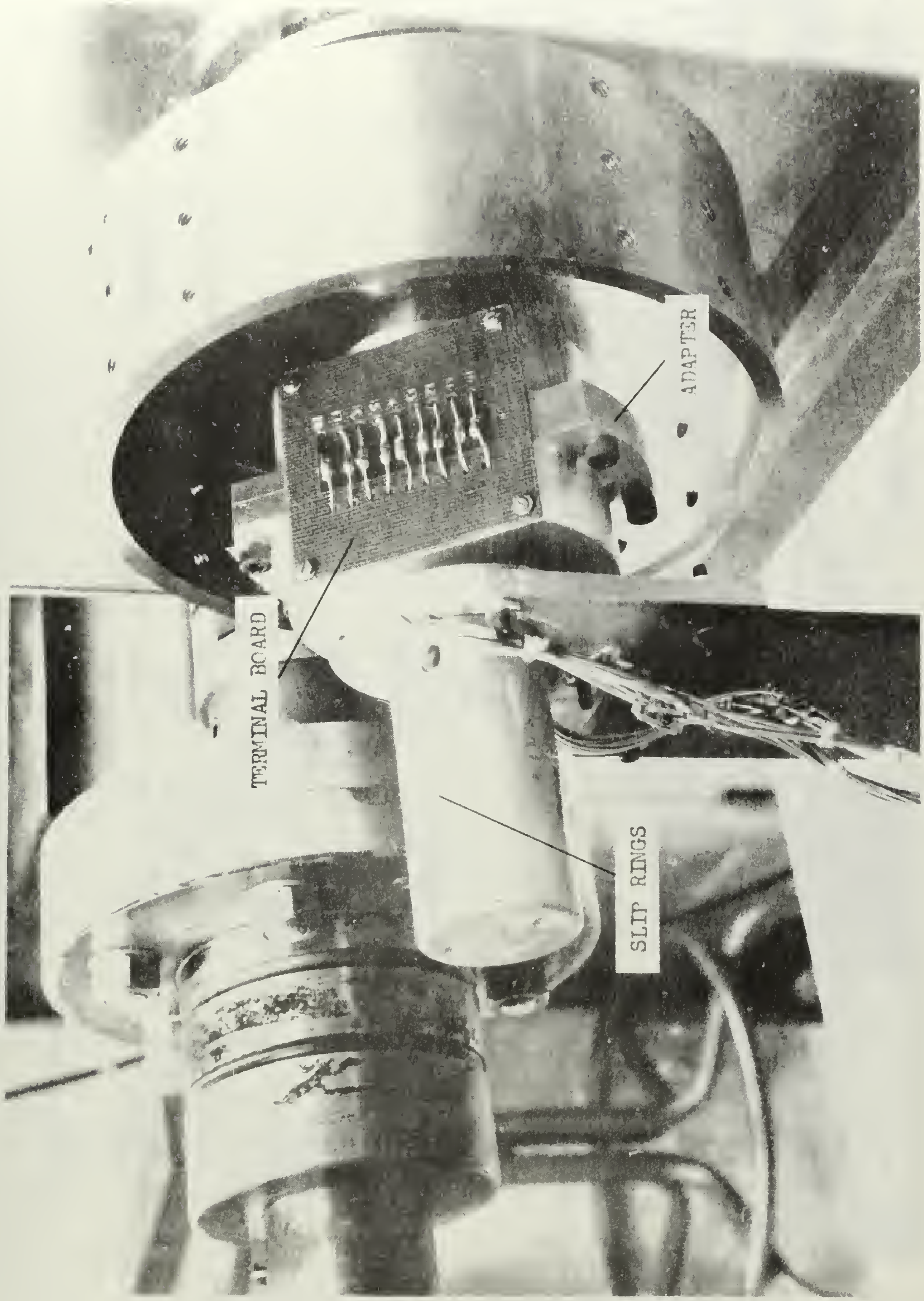
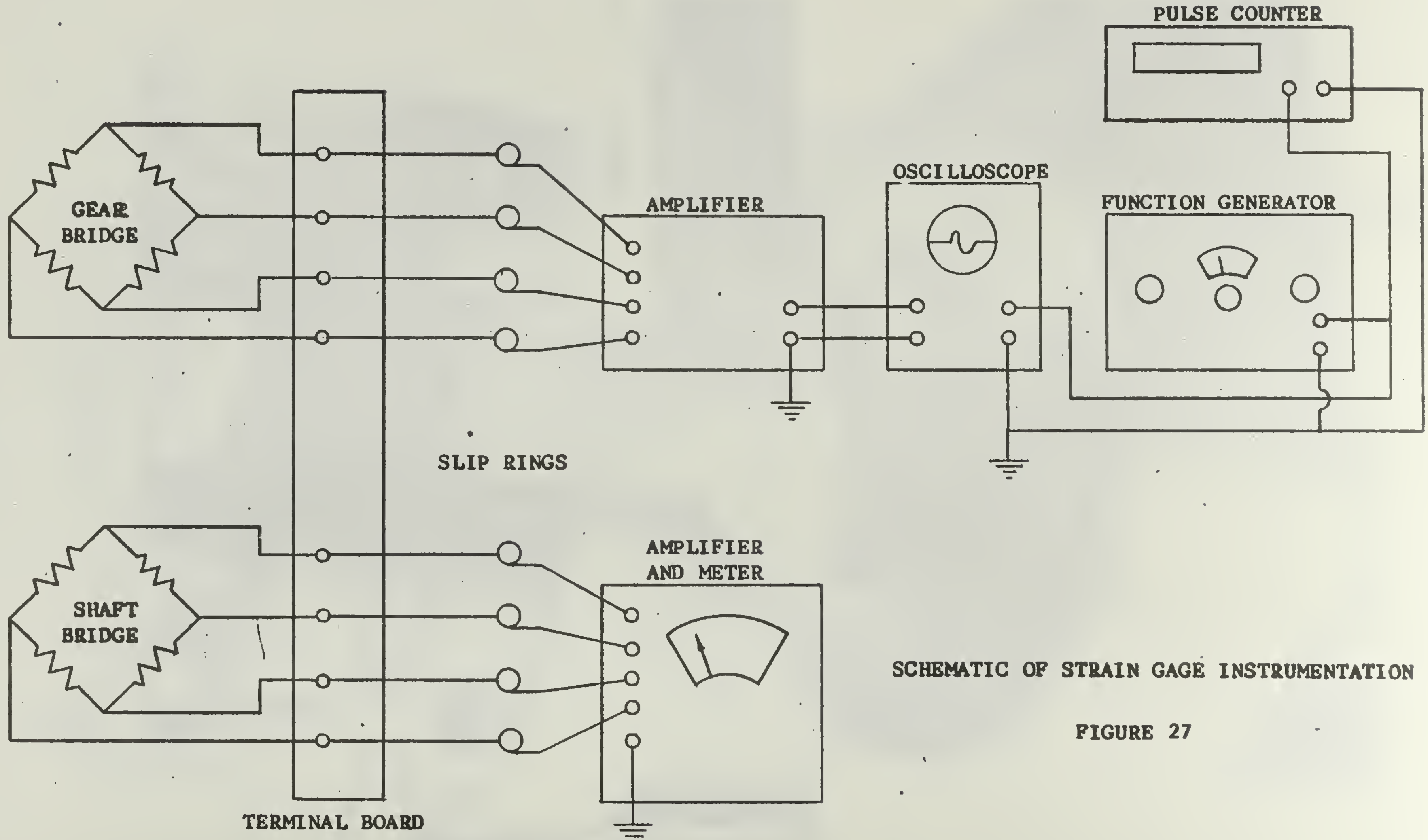


FIGURE 26. SLIP RINGS AND ADAPTER



SCHEMATIC OF STRAIN GAGE INSTRUMENTATION

FIGURE 27

FIGURE 28. STRAIN GAGE INSTRUMENTATION

The transmitted load is measured with a BAM - 1 strain gage bridge amplifier and meter, calibrated to read directly in pounds of transmitted load.

CALIBRATION

The strain gages on the hollow shaft are affected by shear strain. This shear strain must be related to pounds of load in order to calibrate the strain gage amplifier properly.

The following symbols are used for the calculations.

D_i Inside diameter of the hollow shaft.
$D_i = 0.50$ in.

D_o Outside diameter of the hollow shaft.
$D_o = 2.00$ in.

J. Polar moment of inertia of the shaft.

G. Shear modulus
$G = 12,000,000$ psi.

R_p Pitch radius of the test gear.
$R_p = 2.889$ in.

F Gage factor of the strain gages mounted on the shaft.

γ Shear strain.

θ Angle of twist per unit length of shaft.

The torque in a hollow shaft may be related to the angle of twist per unit length of the shaft by [8]

$$\theta = \frac{T}{GJ}$$

where J is the polar moment of inertia of the hollow shaft, given by

$$J = \frac{\pi}{32}(D_o^4 - D_i^4) = 1.565 \text{ in.}^4$$

Shear strain is related to the angle of twist per unit length of the shaft by

$$\gamma = \frac{D_o}{2}$$

Therefore shear strain and torque are related by

$$\gamma = \frac{D_o T}{2GJ}$$

Since torque is a function of transmitted load,

$$T = F_t \times R_p$$

we can relate shear strain directly to transmitted load by

$$\gamma = \frac{D_o \; F_t \; R_p}{2 \; G \; J}$$

or

$$F_t = \frac{2 \; G \; J \; \gamma}{D_o \; R_p}$$

$$= 6.50 \, \gamma$$

The calibration value of shear strain for the BAM - 1 is given by [5]

$$\gamma = \frac{\text{GAGE RESISTANCE}}{\text{GAGE FACTOR}} \times \frac{\text{CALIBRATION SETTING}}{\text{NO. OF WORKING ARMS}}$$

For a calibration setting of one, this gives

$$\gamma = \frac{120}{2.06} \times \frac{1}{4} = 14.56 \text{ microin./in.}$$

Since $F_t = 6.50\gamma$, 14.56 microin./in. of strain corresponds to 94.7 pounds of transmitted load.

With the BAM - 1 adjusted and balanced, the calibration knob is set to 1, then depressed and the gain control set so as to move the meter pointer to read 94.7 on any convenient scale. The meter is then calibrated to read transmitted load in pounds on that scale.

The signals from the instrumented tooth on the test gear are picked up by a BA - 2 bridge amplifier, where from 0 - 3 stages of amplification may be selected. The output from the BA - 2 is fed into the vertical circuit of a cathode ray oscilloscope.

While the BA - 2 provides a means of calibration, and of measuring specific strains, this function is not used since the strains occurring in the vicinity of the strain gages mounted on the test gear tooth are of such a complex nature that a numerical value of strain would be meaningless.

Instead, with all gain controls on both the BA - 2 and the oscilloscope set at a constant value, relative heights and shapes of the wave form are observed.

Speed measurement of the test unit, and stabilization of the gear tooth signal on the oscilloscope are accomplished by the same instrumentation.

In order to observe the signal pulse from the instrumented gear tooth, the oscilloscope horizontal sweep is triggered by a sine wave generator of variable frequency. When the frequency of the sine wave just equals the frequency of the meshing of the instrumented gear tooth, the signal pulse will appear on the oscilloscope once each sweep.

A hand held tachometer is used to determine the approximate speed of the test unit in revolutions per minute. Dividing this by sixty gives revolutions per second. Since the instrumented gear tooth meshes once each revolution, this corresponds to the frequency, in cycles per second, required to properly trigger the oscilloscope.

Conversely, by measuring the triggering frequency of the sine wave generator once the signal has been stabilized, the speed of the test unit can be found to the nearest revolution per minute.

A pulse counter is used in conjunction with a low-frequency function generator to accomplish the triggering and speed measurements.

In operation, two oscilloscopes are used. One is a conventional oscilloscope used for viewing the wave form from the test gear tooth. The other oscilloscope, a Memoscope, is used, in conjunction with an oscilloscope camera, to photograph the wave form.

This photographic method of recording the oscilloscope trace enables the wave form to be analysed at leisure.

EXPERIMENTAL PROCEDURE

The following procedure was used in investigating the loading of the test gear tooth.

The test unit was started according to operating procedures and run at its slowest speed. The unit was then loaded until a desired test load was obtained. This load was held throughout the test run.

Gain and amplification controls were set to produce a wave form from the test gear of a convenient amplitude on the oscilloscope. This wave form corresponds to the load on the gear tooth. When the unit is in an unloaded condition, no signal is received from the test gear.

The speed of the test unit was then varied from its slowest speed up to about 1200 rpm. The wave form was observed and photographed at the slowest test unit speed, and at 100 rpm intervals, starting at 100 rpm.

According to Buckingham's analysis of dynamic loads, at very slow speeds the dynamic load is very nearly equal to the transmitted load. [1] From this it can be reasoned that the amplitude of the wave form, at the pitch point phase of engagement, produced at the test unit's slowest speed corresponds to transmitted load, and that the pitch point amplitude of the wave form at higher speeds is due to the dynamic load.

Other factors are present that can affect the amplitude and shape of the wave form. These are the preceding tooth going out of engagement, the following tooth coming into engagement, and friction forces that are present except at the pitch point, where rolling contact alone takes place.

Since the sweep time of the oscilloscope can be set, and is therefore a known quantity, and the speed of the test unit is known, the beginning and end of test tooth engagement, the pitch point, the disengagement of the preceding tooth, and the engagement of the following tooth can all be located on the wave form.

In order to locate the above points however, it is necessary to know the angle of action, or the angle through which the gear turns from the time a pair of teeth first mesh till that same pair of teeth go out of mesh. This angle is equal to the angle of approach plus the angle of recess, which are the angles from first meshing to the pitch point, and from the pitch point to disengagement, respectively. For mating gears of equal base diameters and equal addenda, which is the case for the test unit, the angle of approach is equal to the angle of recess. [4]

The following symbols are used in the calculations to find the above angles and points.

ϕ Standard pressure angle of gear teeth.
$\phi = 20^{\circ}$

ϕ' Operating pressure angle of gear teeth due to variation in center distance.

C Center distance
$C = 5.80$ in.

D_o Addendum circle diameter. (Same as outside diameter)
$D_o = 5.986$ in.

D_b Base circle diameter.
$D_b = 5.431$ in.

D_p Pitch diameter.
$D_p = 5.779$ in.

Z Length of action.

ψ Angle of action.

ψ_A Angle of approach.

α Angular tooth spacing in degrees.

ω Angular velocity in degrees per second.

It will be noted from the gear data given in appendix I that the center distance for the test gears is greater than the pitch diameter of the test gears. This means that the pitch circles are not tangent to each other, but are separated by a small amount. It also means that the pressure angle will be greater than ϕ . [2]

This pressure angle, the operating pressure angle, is given by

$$\phi' = \cos^{-1}\left[\frac{D_p \cos\phi}{D_o}\right]$$

$$= \cos^{-1}\left[\frac{(5.779)\ (.93969)}{5.80}\right] = 20.55^{\circ}$$

Using this pressure angle, the length of action is given by [4]

$$Z = \left[(D_o^2 - D_b^2)^{\frac{1}{2}}\right] - C \sin\phi'$$

$$= 2.5174 - 2.0359 = 0.4815 \text{ in.}$$

Since the angle of action is made up of the angle of approach and the angle of recess, and these two smaller angles are equal to each other, it follows that the length of approach is equal to the length of recess, and that the sum of the two are equal to the length of action.

The angle of approach, in radians, is given by [4]

$$\psi_A \cong \frac{Z}{D_o}$$

and in degrees by $$\psi_A \cong 57.3 \frac{Z}{D_b} = 5.08^\circ$$

From this, the angle of action is

$$\psi = 2\psi_A = 10.16^\circ$$

Angular tooth spacing is found by simply dividing the number of degrees in a circle by the number of teeth on the test gear.

$$\alpha = \frac{360}{65} = 5.538^\circ$$

To locate the various points of gear action on the wave form, the angular velocity of the test gear is found from the speed of the unit by

$$\omega = \frac{RPM}{60} \times 360^\circ \quad \text{(degrees/sec)}$$

Knowing the angular velocity, and the angle through which any given event occurs, this occurrence can be related to a time measurement.

The abscissa of the oscilloscope trace is in units of time, so any event can be laid off from a reference point on the tooth signal wave form.

The reference point used is where the instrumented tooth goes out of mesh, as this point is the most clearly defined, as may be seen in the following photographs and diagrams.

DISCUSSION OF RESULTS

Figures 29 through 39 present a series of photographs taken for one test run with a transmitted load of 75 pounds. Figures 40 through 43 show enlarged diagrams of the wave form for speeds of 42, 400, 700, and 900 rpm, showing the various points of gear action.

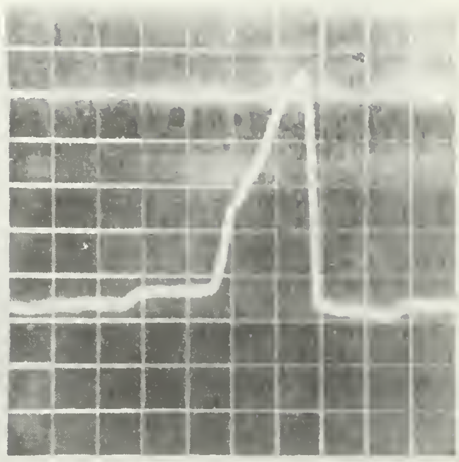


FIGURE 29
SPEED UP RUN
HORIZONTAL SWEEP [illegible]
[illegible]

FIGURE 30
[illegible]
HORIZONTAL SWEEP [illegible]
[illegible] SECONDS

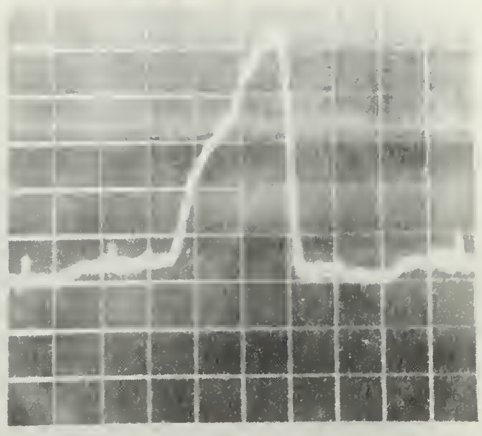


FIGURE 31
SPEED 204 RPM
HORIZONTAL SWEEP 2 MILLISEC/DIV
TRANSMITTED LOAD 75 POUNDS

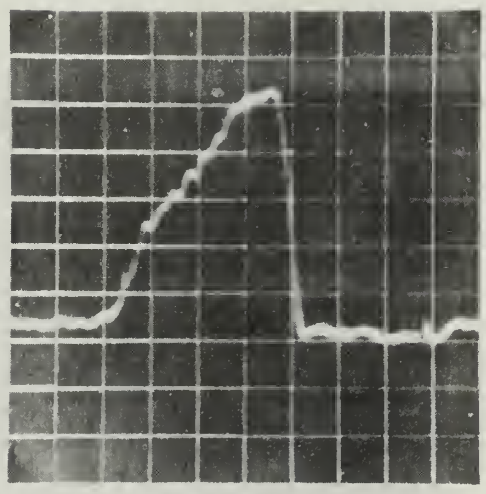


FIGURE 32
SPEED 300 RPM
HORIZONTAL SWEEP 900 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

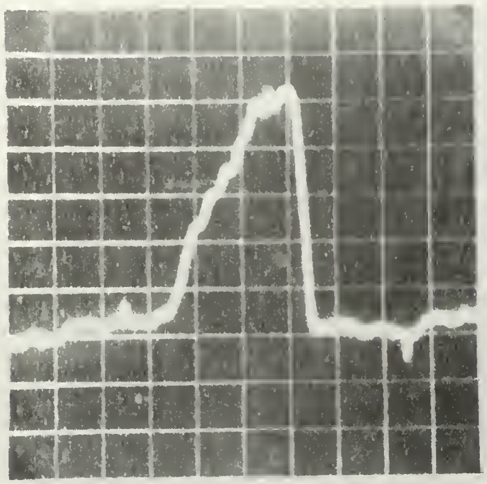


FIGURE 33
SPEED [illegible]
HORIZONTAL SWEEP 200 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

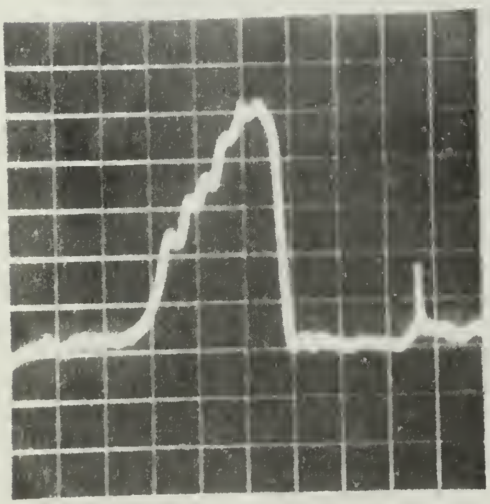


FIGURE 34
SPEED 50%
HORIZONTAL SWEEP 200 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

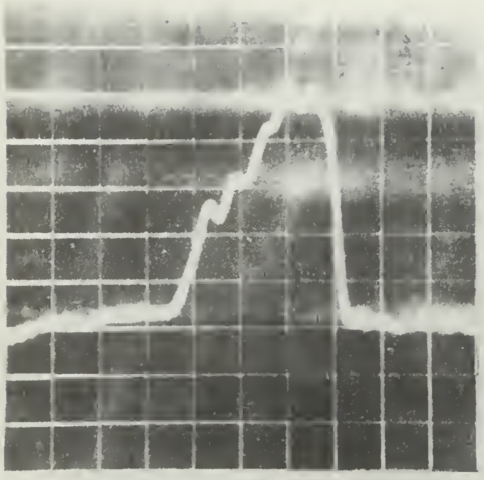


FIGURE 35
SPEED 606 RPM
HORIZONTAL SWEEP 500 MICRO
TRANSMITTED LOAD 75 POUNDS

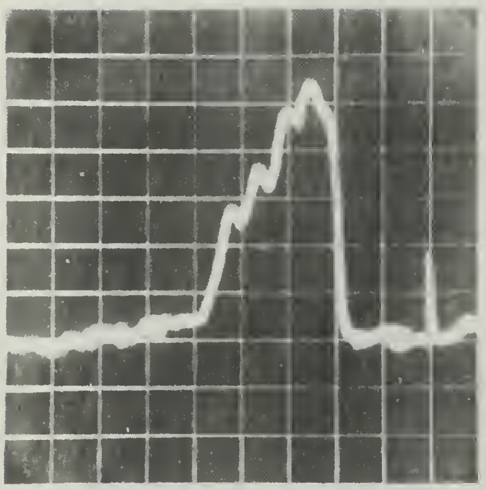


FIGURE 36
SPEED 708 RPM
HORIZONTAL SWEEP 500 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

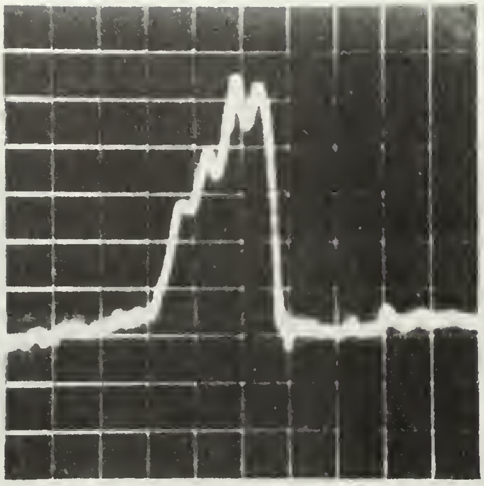


FIGURE 37
SPEED 810 RPM
HORIZONTAL SWEEP 500 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

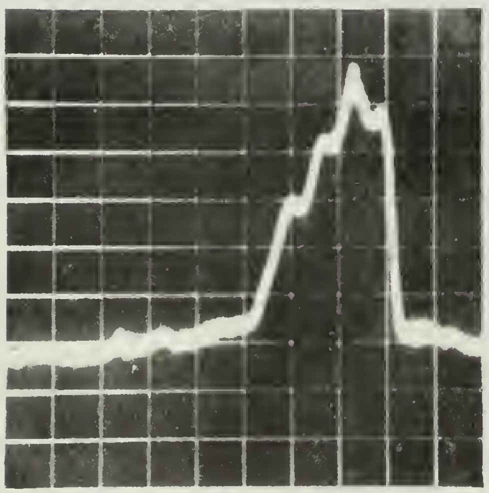


FIGURE 38
SPEED 900 RPM
HORIZONTAL SWEEP 400 MICROSEC/DIV
TRANSMITTED LOAD 75 POUNDS

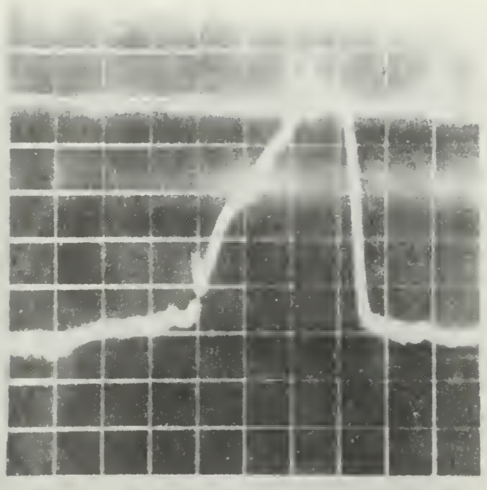
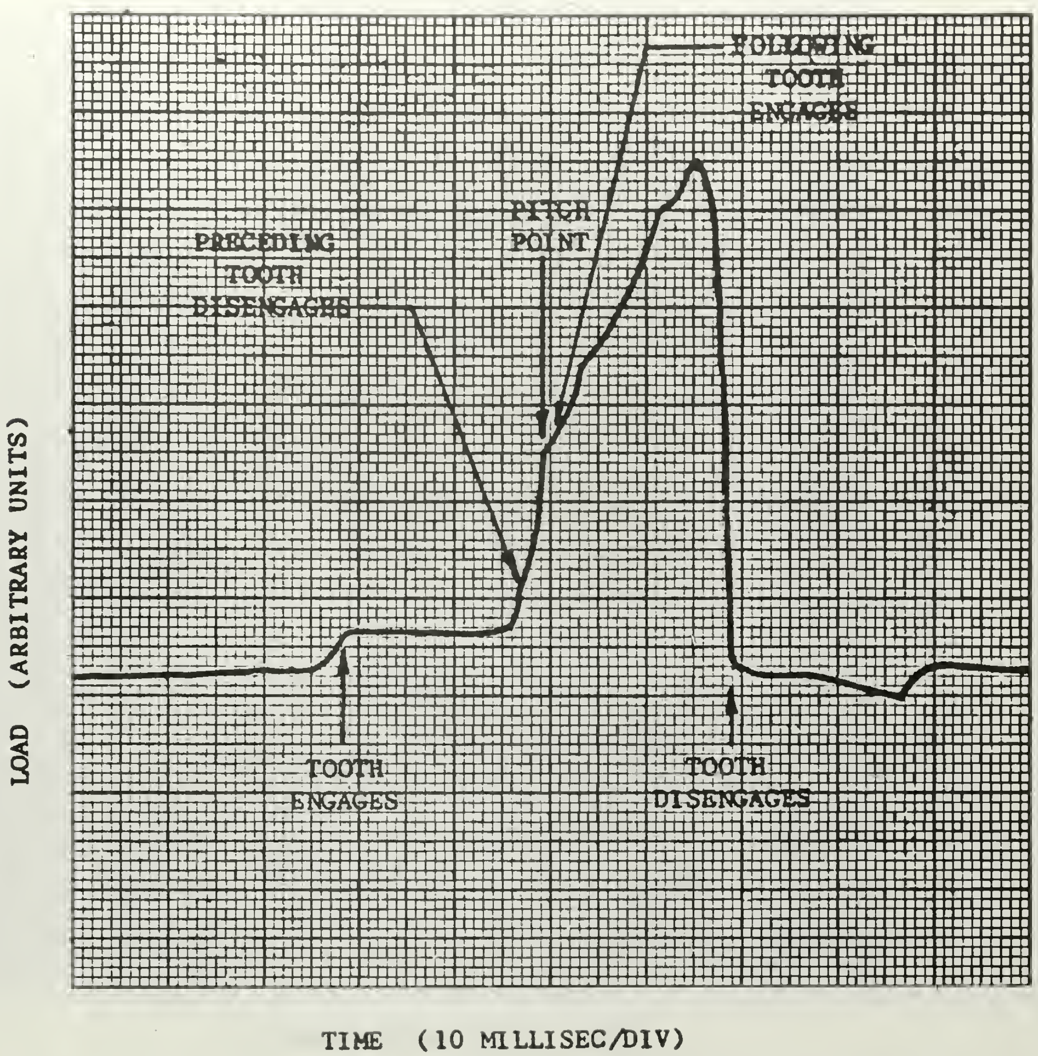


FIGURE 30
SPEED 1002 RPM
HORIZONTAL SWEEP 300 [illegible]
TRANSMITTED LOAD 75 POUNDS



Enlarged drawing of photographed waveform. Test unit speed is 42 RPM. Transmitted load is 75 pounds. Note the small jump in the trace as the tooth begins it's engagement. Note also the change in the slope of the trace as the following tooth engages. This is an indication that the following tooth has taken part of the load and that the rate of loading of the test tooth has been decreased.

FIGURE 40

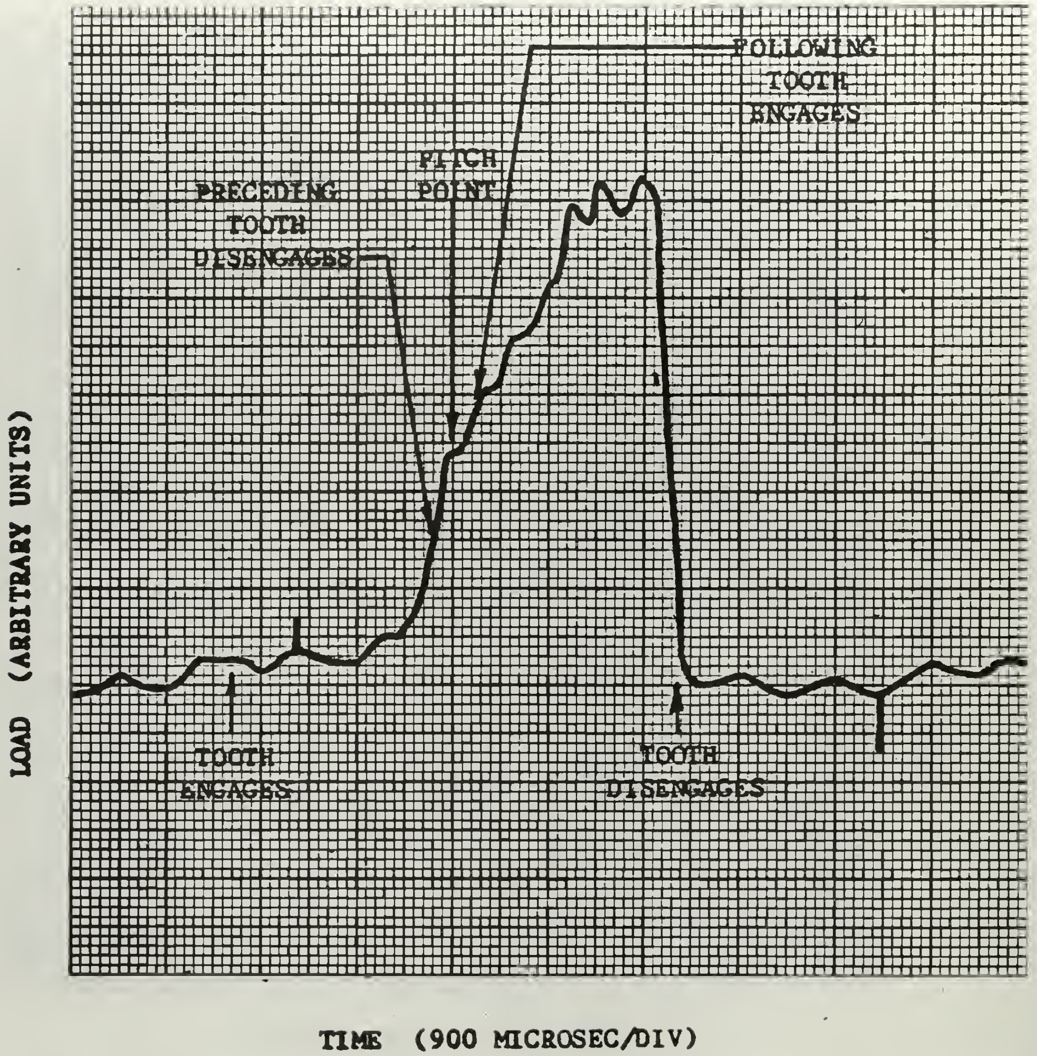
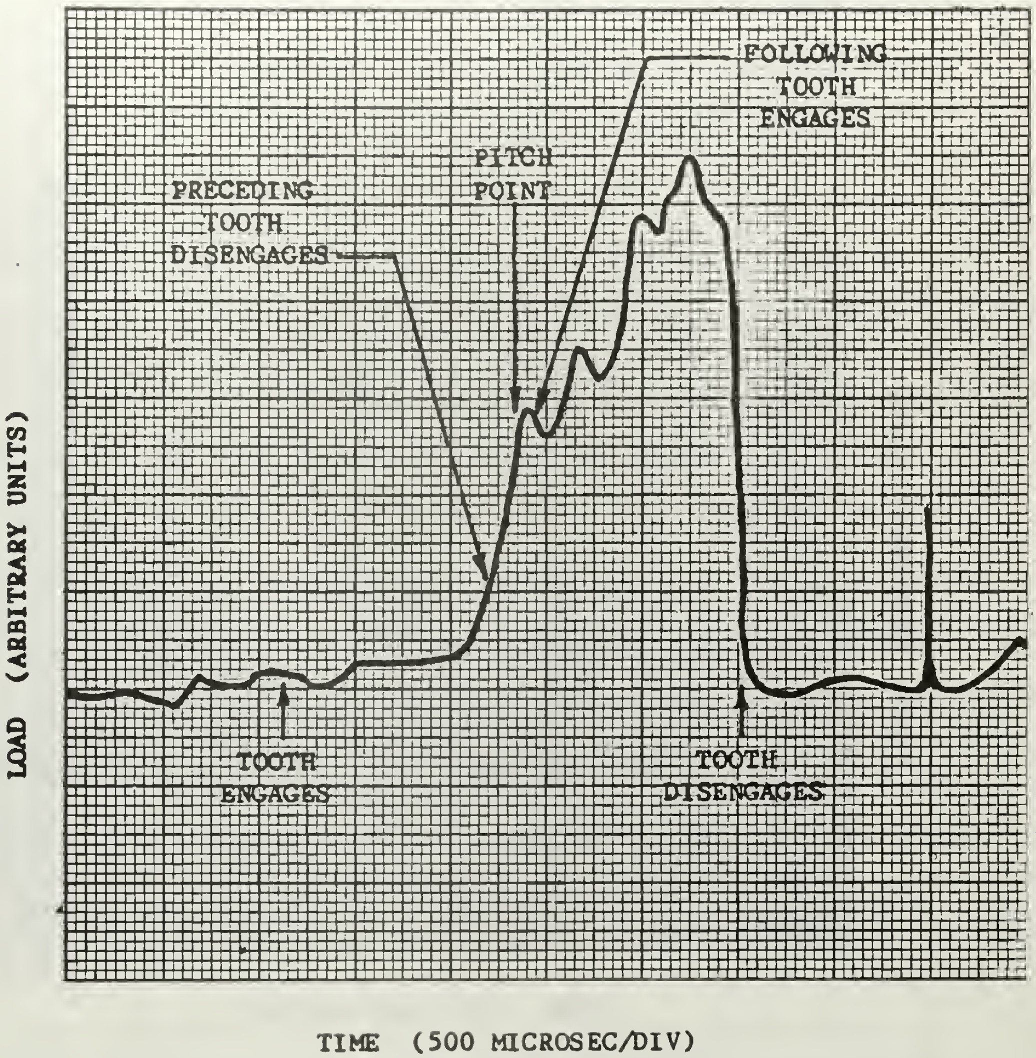


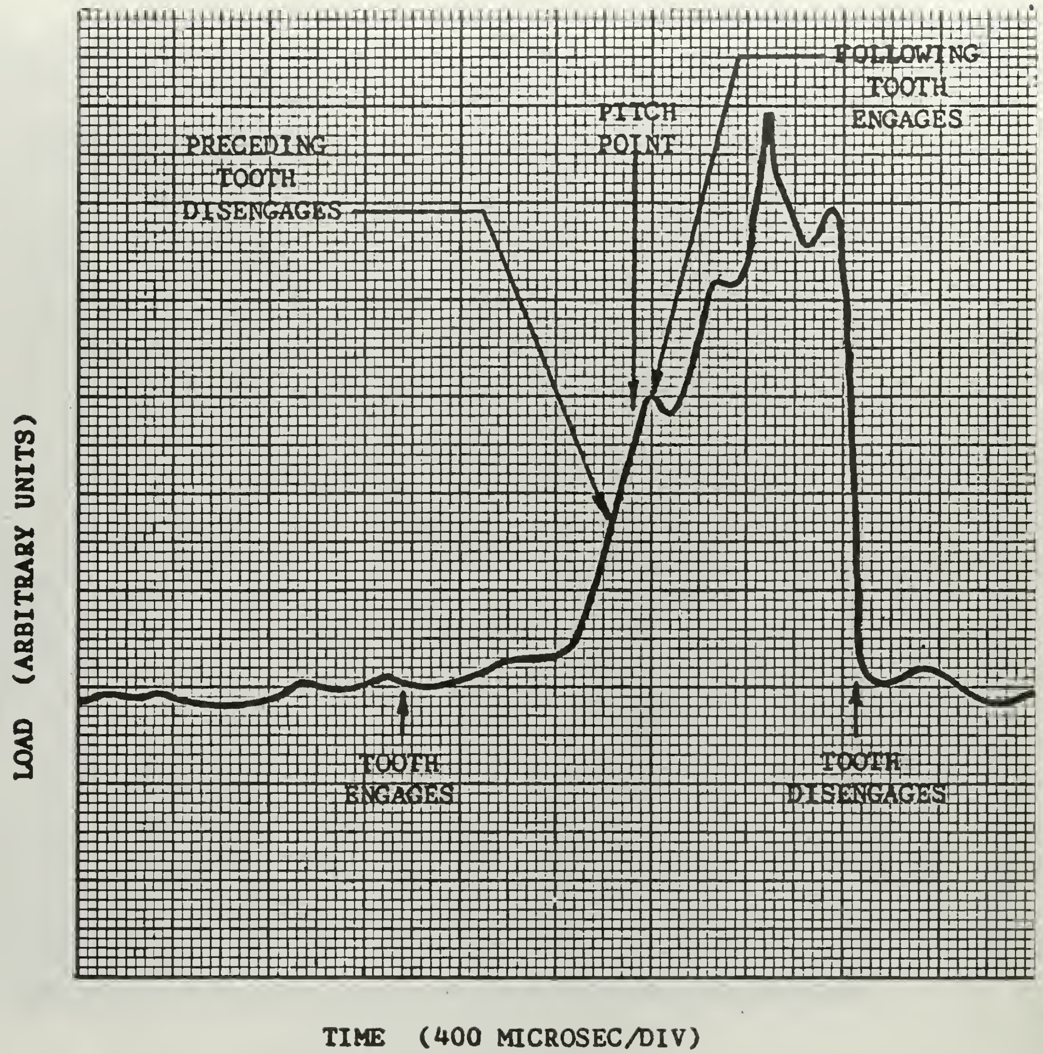
Diagram of waveform produced at 402 RPM, with a transmitted load of 75 pounds.. The begining of tooth engagement is less noticeable than it was for 42 RPM. The indication of the following tooth is more pronounced however.

FIGURE 41



Enlarged waveform at 708 RPM and 75 pounds transmitted load. Tooth engagement is virtually undescernible. The following tooth engagement now produces a negative slope, indicating a momentary reduction in tooth load.

FIGURE 42



Waveform at 900 RPM with 75 pounds transmitted load. Again initial tooth engagement is undescernible. The following tooth engagement produces a reduction in tooth loading. The preceding tooth going out of mesh produces no noticeable changes to the trace.

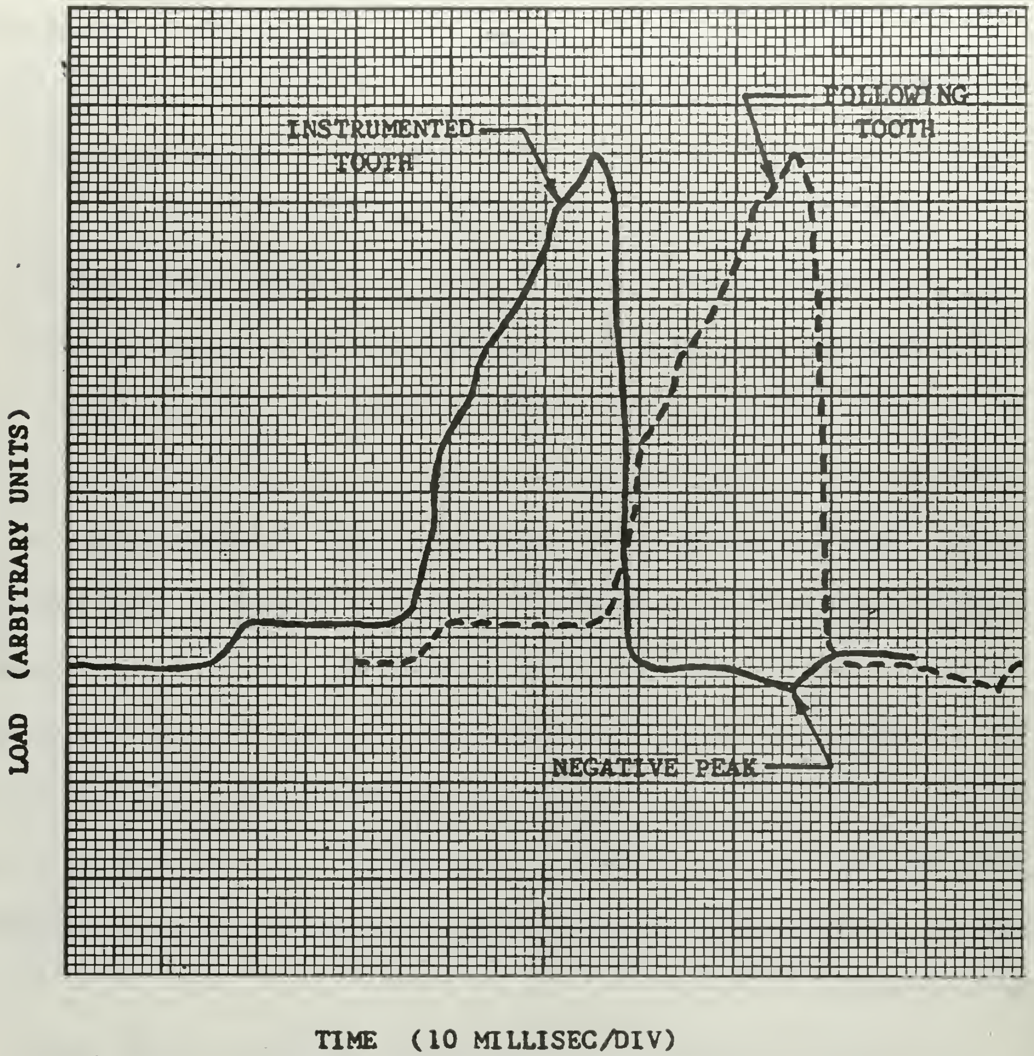
FIGURE 43

As the speed of the test unit is increased, it will be noted that the signal trace before and after tooth engagement becomes less well defined than it is for low speeds. This is due to slip ring noise increasing with speed. The trace of the tooth engagement itself remains fairly well defined at all speeds.

A phenomenon that is most clearly observable in the trace photograph obtained at 42 rpm is what appears to be an overshoot of the trace as the instrumented tooth goes out of engagement. Figure 44 shows that this is due to the action of the following tooth. By superimposing the trace on itself, but displaced one tooth width timewise, a representation is obtained of what the loading on two successive teeth would look like. It can be seen that the overshoot, with its small peak, corresponds exactly to the loading of the following tooth. This means that the load on the following tooth affects the strain gage in the opposite manner, but to a lesser extent, from which it is affected by the load on the instrumented tooth itself.

It will be noticed from the enlarged diagrams of the trace photographs, figures 40 through 43, that there is very little indication when tooth engagement starts. This can be attributed to the placement of the strain gages on the gear tooth, figure 23. The load first comes on the tooth at a point where there is little or no effective lever arm acting on the strain gage, and it is not until the loading point moves along the face of the tooth, toward the tip, that the strain gage sends a significant signal.

It is recommended that in future use of the testing maching various orientations of the strain gages on the gear tooth be tried,



A second tooth engagement waveform is projected on the test tooth waveform, indicating what two successive tooth engagements would look like. This illustrates the effect the following tooth has on the strain gage mounted on the test tooth. Note that the loading peak of the following tooth and the small negative peak of the instrumented tooth coincide.

FIGURE 44

attempting to find a position that is less sensitive to the load on other teeth, and one that gives a stronger indication of initial tooth engagement.

The wave forms of figures 40 through 43 have several changes in slope that occur after the pitch point and the engagement of the following tooth. The large peak is where the loading point is just reaching the tip of the tooth, and the tooth is beginning to go out of mesh. The other phenomena, which appear as nodes much the same size and shape as the one coinciding with the pitch point and following tooth engagement, are not explainable in terms of the meshing action of the test gears.

It has been suggested that the gear teeth vibrate after the dynamic load comes on them. [9] On the other hand, in a four square device, such as the test unit is, there may be complex dynamic loadings occurring from the action of the helical loading and drive gears.

Whatever the cause, it can not be explained at this time, and should be the subject of future investigations.

The amplitude of the trace at the pitch point was measured for each speed recorded. This amplitude, divided by the pitch point amplitude of the trace obtained at the slowest test unit speed, gives the ratio of dynamic load to transmitted load. These results are plotted and shown in figure 45, which is a repeat of figure 22, showing the predicted ratios obtained from Buckingham's average and exact equations.

It is felt that Buckingham's equations do not hold for low powers, [2] and since the power input to the test unit did not exceed six horsepower, a load ratio curve obtained from

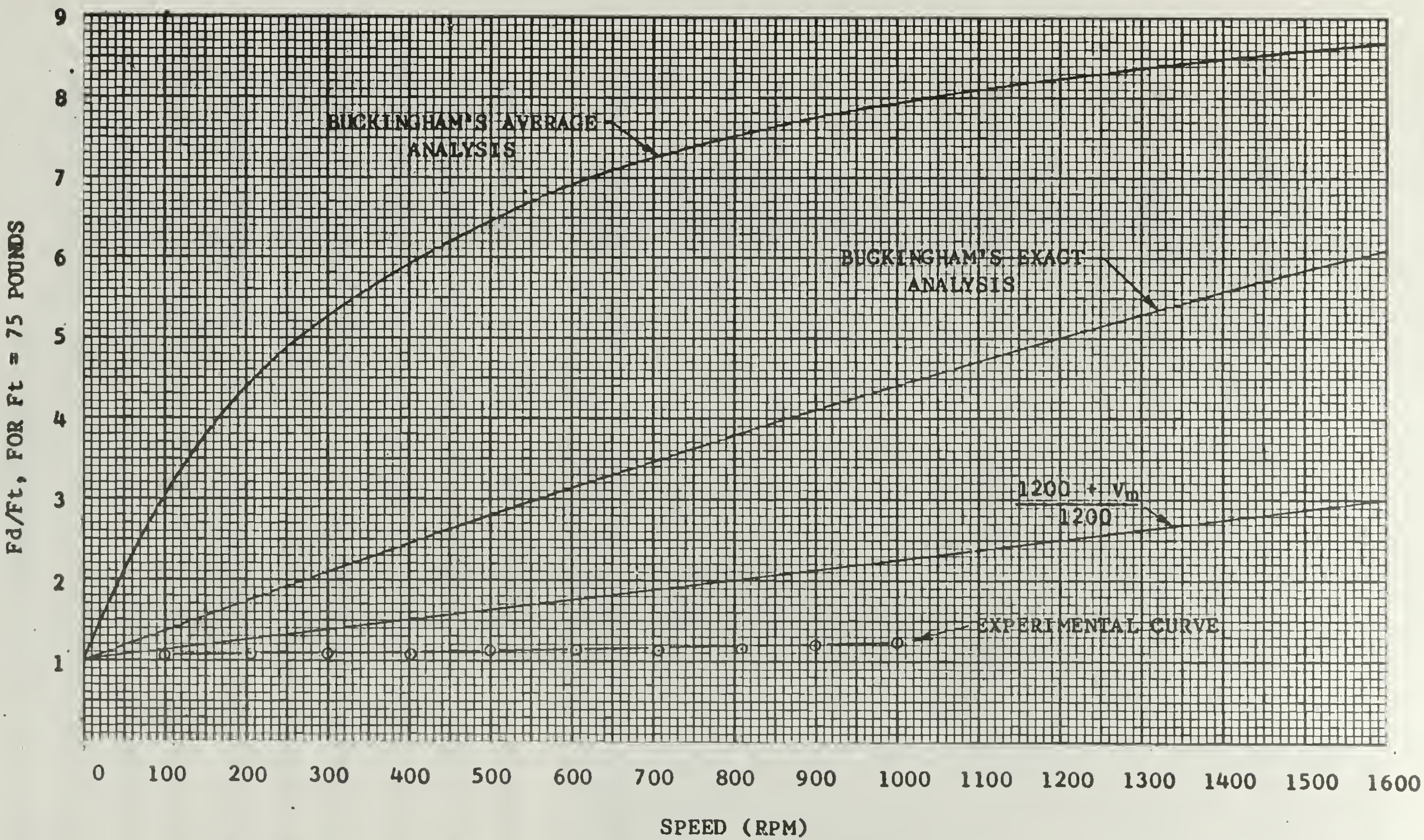


FIGURE 45, SHOWING DYNAMIC LOADING CURVE OBTAINED BY EXPERIMENT, COMPARED TO THREE THEORETICAL CURVES.

$$F_d = \frac{1200 + V_m}{1200} F_t$$

is also shown. The use of this equation is recommended by Faires for small power transmission.

The experimental curve lies well below all the other curves, indicating an almost total lack of dynamic loading.

This indication of dynamic loads well below the predicted values might be attributed to the low input power, but it is felt that it is a result of the design of the test gears themselves. The dynamic loading occurs in the vicinity of the pitch point. It can be seen from figures 40 through 43 that the following tooth begins its engagement also in the vicinity of the pitch point. In fact the two events almost coincide. This means that just as a heavy load is due to come on the instrumented tooth, the following tooth takes part of this load as it comes into mesh.

The number of gear teeth that are in mesh at any given time is a function of addendum circle diameter, base circle diameter, center distance, pressure angle, and diametral pitch. [2] Using these parameters, a contact ratio, which can be thought of as an average number of gear teeth that are in contact while the gears are meshing, may be computed.

For any given velocity ratio, some or all of the above parameters can be varied to give a contact ratio that would produce the condition where the following tooth engages at the time the dynamic load is coming on a tooth, namely the pitch point.

V. CONCLUSIONS AND RECOMMENDATIONS

The test machine was designed and built to investigate the loading of gear teeth, using strain gages. In its first phase of operation, low speeds and low loading, it has proved to be successful. The load on an individual gear tooth was detected and recorded under dynamic conditions.

Possibly the most important aspect of the work with the test unit was the lack of correlation between observed loads and predicted loads. The discovery that the following tooth was beginning its engagement when the instrumented tooth was at its pitch point phase of engagement, and, in effect, sharing the dynamic load of the instrumented tooth, indicates that it may be possible to design out dynamic load effects on individual gear teeth by suitable adjustment of the contact ratio.

Much work remains to be done, however. The placement of the strain gages on the gear teeth should be investigated further in order to achieve optimum signals during the entire tooth engagement.

The mass moment of inertia of the system should be varied to verify if the test unit shafting is, in fact, flexible enough to cause any attached masses to have no effect on gear loading. This capability is built into the test unit, but time did not permit its use.

Lubricants and lubricant temperature should be varied. Lubrication films between the gear teeth undoubtedly have some effect.

The speed and loading range must be expanded before any really conclusive results can be obtained. This will probably require modification of the test machine to obtain greater input powers.

Another set of test gears, ones having a contact ratio such that pitch point loading is not complicated by the action of other teeth, should be obtained.

These are but a few of the considerations to be made in carrying on with future testing and use of the gear testing machine.

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APPENDIX

GEAR DATA

	TEST DRIVE GEAR	TEST GEAR
FACE WIDTH	0.5625	0.375
PITCH DIAMETER	5.7795	5.7795
BASE CIRCLE DIAMETER	5.4309	5.4309
OUTSIDE DIAMETER	5.9877 5.9847	5.9877 5.9847
WHOLE DEPTH	0.2083	0.2083
NORMAL CIRCULAR TOOTH THICKNESS	0.1440 0.1425	0.1440 0.1425
TOOTH ERROR	0.001	0.001
BACKLASH WITH MATE	.003/.005	.003/.005
PRESSURE ANGLE	20°	20°
NUMBER OF TEETH	65	65

The tooth error was estimated from the following data furnished by the gear manufacturer.

.0003 in. pitch error

.0004 in. involute profile error

.0003 in. lead error

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13. ABSTRACT

The action of mating gear teeth is a very complex thing. Very little is known of the forces that act on gears as they mesh. It is with this in mind that a testing machine was designed, built, and used to investigate the feasibility of obtaining gear tooth load information, by the use of strain gages mounted on the teeth of a test gear. The construction and operation of the testing machine is described, followed by the results of experiments conducted. The application of the experimental results to some design considerations in gears is then discussed. A great need exists for gear loading information, and the gear testing machine has proved to be successful in the initial phases of its operation.

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